19 Parity, Charge Conjugation and CP

19.1 Intrinsic Parity

In the same way that nuclear states have parity, so hadrons, which are bound states of quarks (and antiquarks) have parity. This is called intrinsic parity, η and under a parity inversion the wavefunction for a hadron acquires a factor η

$$P\Psi_{\{P\}}(\mathbf{r}) \;==\; \Psi_{\{P\}}(-\mathbf{r}) \;=\; \eta_{\{P\}}\Psi_{\{P\}}(\mathbf{r}).$$

η can take the values ± 1 noting that applying the parity operator twice must bring us back to the original state.

The lighter baryons (for which there is zero orbital angular momentum) have positive intrinsic parity.

On the other hand, antiquarks have the opposite parity from quarks. This means that the light antibaryons have negative parity. It also means that the light mesons, such as pions and kaons, which are bound states of a quark and an antiquark have negative intrinsic parity. The lightest spin-one mesons, such as the ρ-meson, also have zero orbital angular momentum and thus they too have negative intrinsic parity - they have spin-one because of the alignment of the spins of the (valence) quark and antiquark.

For more massive (higher energy) particles, the quarks can be in non-zero orbital angular momentum states so that both baryons and mesons with higher masses can have either parity.

Parity is always conserved in strong interaction processes. A consequence of this is the decay

$$\rho^0 \;\rightarrow\; \pi^+ + \pi^-$$

Since ρ mesons have spin-one and pions have spin zero the final pion state must have $l = 1$. The ρ has negative intrinsic parity and so do the two pions. The orbital angular momentum $l = 1$ means that the parity of the final state is

$$\eta_\pi^2(-1)^1 \;=\; -1,$$

so that parity is conserved. On the other hand, two π^0's cannot be in an $l = 1$ state. The reason for this is that pions are bosons and so the wavefunction for two identical pions must be symmetric under interchange, whereas the wavefunction for an $l = 1$ state is antisymmetric if we interchange the two pions. This means that the decay mode

$$\rho^0 \;\rightarrow\; \pi^0 + \pi^0$$

is forbidden.

If we look at the non-leptonic weak decay of a K^+ into pions (weak because strangeness is not conserved) we find that

$$K^+ \;\rightarrow\; \pi^+ + \pi^0$$

and

$$K^+ \rightarrow \pi^+ + \pi^0 + \pi^0, \quad \text{or} \quad \pi^+ + \pi^+ + \pi^-$$

both occur. Since the K^+ has negative parity and zero spin - so that the final state cannot have any orbital angular momentum, the final two pion state has even parity, whereas the final three pion state has odd parity.

This is a demonstration that the weak interactions do not conserve parity - and this was in fact observed before C.S. Wu's experiment on the β-decay of ^{60}Co.

19.2 Charge Conjugation

This is the operation of replacing particles by their antiparticles

$$C\Psi_{\{P\}} = \Psi_{\{\overline{P}\}}$$

e.g.

$$C\Psi_{\pi^+} = \Psi_{\pi^-}$$
$$C\Psi_p = \Psi_{\overline{p}}$$

Some mesons are their own antiparticles such as a π^0 or the J/Ψ (a quark-antiquark pair of the same flavour). In this case we have a charge conjugation quantum number η^C

$$C\Psi_{\pi^0} = \eta^C \Psi_{\pi^0},$$

where η^C can take the values ± 1 - again noting that the application of the charge conjugation operator twice must bring us back to the original state.

A photon has charge conjugation $\eta^C = -1$. This is because under charge conjugation electric charges switch sign and therefore so do electric and magnetic fields. We know that the π^0 can decay into two photons via electromagnetic interaction, which are invariant under charge conjugation

$$\pi^0 \rightarrow \gamma + \gamma$$

This forces the charge conjugation of π^0 to be $\eta^C = +1$.

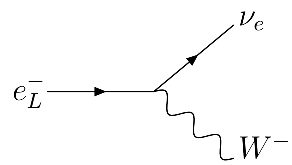
The spectra of charmonium ($c - \bar{c}$ bound states), or bottomonium ($b - \bar{b}$ bound states) contain both positive and negative charge conjugation states.

19.3 CP

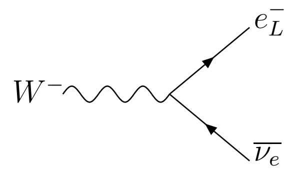
Like parity, charge conjugation is conserved by the strong and electromagnetic interactions but *not* by the weak interactions,

On the other hand, the weak interactions are (almost) invariant under the combined operations of charge conjugation *and* parity inversion, known as "CP".

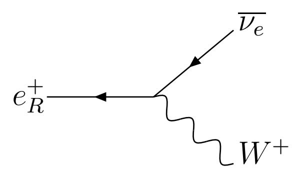
Thus the weak interactions will allow a (highly relativistic) left-handed (negative helicity) electron to convert into a neutrino emitting a W^-



(or alternatively a W^- will decay into a left-handed electron and an antineutrino)



Similarly a right-handed (positive helicity) positron can convert into an antineutrino



If it were possible to repeat the experiment of C.S. Wu using the antiparticle of ^{60}Co, $\overline{^{60}\mathrm{Co}}$ which decays into $\overline{^{60}\mathrm{Ni}}$ emitting positrons and neutrinos, one would find that the positrons tended to be emitted in the same direction as the spin of the antinucleus (whereas in the original experiment they tended to be emitted in the opposite direction from the spin of the nucleus).

19.4 $K^0 - \overline{K^0}$ Oscillations

The invariance of the weak interactions under CP has consequences for the K^0 and $\overline{K^0}$ particles (and also for B^0 and $\overline{B^0}$ mesons currently being studied at the BaBar collaboration at SLAC.)

$$P\Psi_{K^0} = -\Psi_{K^0}$$

and

$$C\Psi_{K^0} = \Psi_{\overline{K^0}}$$

so that

$$CP\Psi_{K^0} = -\Psi_{\overline{K^0}}.$$

This means that the 'particles' K^0 and $\overline{K^0}$ are not eigenstates of CP. But if CP is conserved, then the energy eigenstates (i.e. masses) must also be eigenstates of CP (CP commutes with the Hamiltonian). These eigenstates of CP are

$$\Psi_{K_L} = \frac{1}{\sqrt{2}}\left(\Psi_{K^0} + \Psi_{\overline{K^0}}\right), \quad \mathrm{CP} = -1$$

and

$$\Psi_{K_s} \;=\; \frac{1}{\sqrt{2}}\left(\Psi_{K^0} \,-\, \Psi_{\overline{K^0}}\right), \quad \mathrm{CP} = +1$$

where L and S stand for long and short for reasons we shall see. These mass eigenstates are therefore not pure K^0 or $\overline{K^0}$ states, but quantum superpositions of the two.

The allowed non-leptonic decays of these states are

$$K_L \;\rightarrow \pi^0 \,+\, \pi^0 \,+ \pi^0, \quad \text{or} \quad \pi^0 \,+\, \pi^+ \,+ \pi^-,$$

because there is no orbital angular momentum as the kaons and pions both have spin zero and we require three pions to make a $CP = -1$ state because the pions have negative parity. Likewise we have

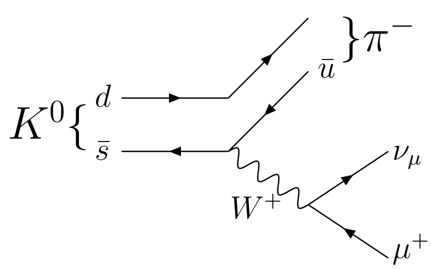
$$K_S \;\rightarrow \pi^0 \,+\, \pi^0, \quad \text{or} \quad \pi^+ \,+\, \pi^-,$$

because two pions give us a $CP = +1$ state.

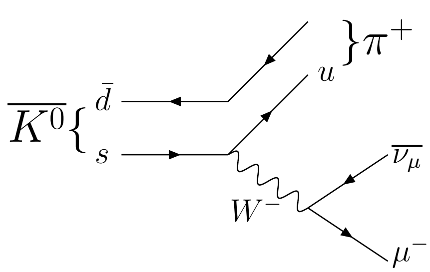
The lifetime of the K_S is shorter than that of the K_L ($\tau_S = 10^{-10}$ s. compared with $\tau_L = 10^{-8}$ s), because the Q-value for the decay into only two pions is larger than that for a decay into three pions ($m_K - 2m_\pi \,>\, m_K - 3m_\pi$).

On the other hand, we can distinguish a K^0 ($\bar{s} - d$ bound state) from a $\overline{K^0}$ ($s - \bar{d}$ bound state) by their semi-lepton decay modes

$$K^0 \;\rightarrow\; \pi^- \,+\, \mu^+ \,+\, \nu_\mu$$



$$\overline{K^0} \;\rightarrow\; \pi^+ \,+\, \mu^- \,+\, \overline{\nu_\mu}$$



If, at time $t = 0$, we have a pure K^0, this is a superposition of the K_L and K_S states

$$\Psi_{K^0}(t = 0) \;=\; \frac{1}{\sqrt{2}}\left(\Psi_{K_L} \,+\, \Psi_{K_S}\right).$$

The K_L and K_S have slightly different masses

$$\frac{\Delta m}{m} = 7 \times 10^{-15}$$

K_L and K_S therefore have different energies, which means that their wavefunctions have different frequencies.

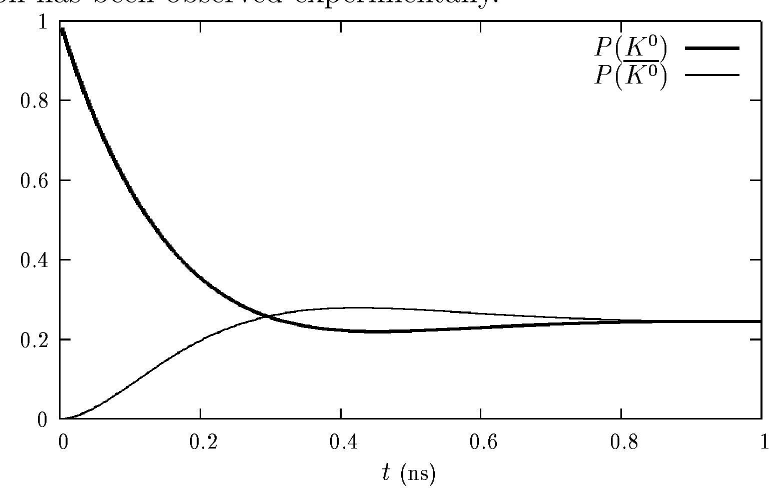
Applying the Schrödinger equation to obtain the time dependence of the wavefunction, whgich at time $t = 0$ represents a pure K^0 state, we obtain a wavefunction which contains oscillations between the wavefunction for a K^0 and the wavefuntion for a $\overline{K^0}$, so that if at some later time t the particle decays semi-leptonically the probabilities $P(K^0)$ or $P(\overline{K^0})$ of observing a K^0 decay (decay products $(\mu^+, \pi^- \nu_\mu)$) or $\overline{K^0}$ decay (decay products $(\mu^-, \pi^+ \overline{\nu_\mu})$) are of the form

$$P(K^0) = A(t) + B(t) \cos\left(\Delta m c^2 t/\hbar\right)$$
$$P(\overline{K^0}) = A(t) - B(t) \cos\left(\Delta m c^2 t/\hbar\right),$$

where $\Delta m = m_{K_L} - m_{K_S}$.

In other words, as time progresses there are oscillations between the K^0 and $\overline{K^0}$ states (details of the calculation are shown in the Appendix).

This oscillation has been observed experimentally.



This is a striking example of the effects of quantum interference.

In 1964, Christensen et.al. observed a few decays of K_L into two pions. Such a decay, in which the $(CP = -1)$ K_L decays into a $CP = +1$ final state, indicated that CP invariance was violated to a very small extent by the weak interactions.

19.5 Summary of Conservation laws

- **Baryon number:** baryons=+1, antibaryons=-1, mesons, leptons=0.
- **Lepton number:**
  - electron number: e^-, $\nu_e = 1$, e^+, $\overline{\nu_e} = -1$
  - muon number: μ^-, $\nu_\mu = 1$, μ^+, $\overline{\nu_\mu} = -1$
  - τ number: τ^-, $\nu_\tau = 1$, τ^+, $\overline{\nu_\tau} = -1$

	Strong Interactions	**Electromagnetic Interactions**	**Weak Interactions**
Baryon number	yes	yes	yes
Lepton number (all)	yes	yes	yes
Angular momentum	yes	yes	yes
Isospin	yes	no	no
Flavour	yes	yes	no
Parity	yes	yes	no
Charge conjugation	yes	yes	no
CP	yes	yes	almost

Appendix
Neutral Kaon Oscillations

If at time $t = 0$, we prepare a state which is pure K^0 (e.g. a decay product of a strongly decaying particle with strangeness +1), then in terms of the wavefunctions for K_L and K_S the wavefucntion at time $t = 0$ is

$$\Psi(t=0) \ = \ \frac{1}{\sqrt{2}} \left(\Psi_{K_L}(t=0) \ + \ \Psi_{K_S}(t=0) \right).$$

The wavefunctions for K_L and K_S therefore have different oscillatory time dependences

$$\exp\left(-im_{K_L}c^2t/\hbar\right) \quad \text{and} \quad \exp\left(-im_{K_S}c^2t/\hbar\right)$$

as well as exponentially decaying time dependent factors

$$\exp\left(-t/(2\tau_L)\right) \quad \text{and} \quad \exp\left(-t/(2\tau_S)\right),$$

indicating that the probabilities of the particles surviving at time t are

$$\exp\left(-t/\tau_L\right) \quad \text{and} \quad \exp\left(-t/\tau_S\right) \quad \text{respectively}.$$

The time dependence of Ψ_{K_L} and Ψ_{K_S} are given by

$$\Psi_{K_L}(t) \ = \ \Psi_{K_L}(t=0)\exp\left(-im_{K_L}c^2t/\hbar\right)\exp\left(-t/(2\tau_L)\right)$$

$$\Psi_{K_S}(t) \ = \ \Psi_{K_S}(t=0)\exp\left(-im_{K_S}c^2t/\hbar\right)\exp\left(-t/(2\tau_S)\right)$$

Therefore the above wavefunction at time t may be written

$$\Psi(t) \ = \ \frac{1}{\sqrt{2}}\left\{\Psi_{K_L}\exp\left(-im_{K_L}c^2t/\hbar - t/(2\tau_L)\right) + \Psi_{K_S}\exp\left(-im_{K_S}c^2t/\hbar - t/(2\tau_S)\right)\right\}$$

Writing this out in terms of wavefunctions for K^0 and $\overline{K^0}$ we get

$$\begin{aligned}\Psi(t) \ &= \ \frac{1}{2}\left\{\Psi_{K^0}\left[\exp\left(-im_{K_L}c^2t/\hbar - t/(2\tau_L)\right) + \exp\left(-im_{K_S}c^2t/\hbar - t/(2\tau_S)\right)\right]\right. \\ &+ \ \left.\Psi_{\overline{K^0}}\left[\exp\left(-im_{K_L}c^2t/\hbar - t/(2\tau_L)\right) - \exp\left(-im_{K_S}c^2t/\hbar - t/(2\tau_S)\right)\right]\right\}\end{aligned}$$

The modulus squared of the coefficients of Ψ_{K^0} and $\overline{\Psi_{K^0}}$ are the probabilities that at time t the particle is a K^0 or $\overline{K^0}$, respectively. These probabilities are

$$P(K^0) \ = \ \frac{1}{4}\left[\exp\left(-t/\tau_L\right) + \exp\left(-t/\tau_S\right) + 2\exp\left(-t(\tau_L+\tau_S)/(\tau_L\tau_S)\right)\cos\left(\Delta mc^2t/\hbar\right)\right]$$

and

$$P(\overline{K^0}) \ = \ \frac{1}{4}\left[\exp\left(-t/\tau_L\right) + \exp\left(-t/\tau_S\right) - 2\exp\left(-t(\tau_L+\tau_S)/(\tau_L\tau_S)\right)\cos\left(\Delta mc^2t/\hbar\right)\right]$$

where $\Delta m = m_{K_L} - m_{K_S}$.