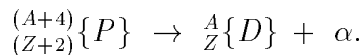


6 Alpha Decay

α -decay is the radioactive emission of an α -particle which is the nucleus of ${}^4_2\text{He}$, consisting of two protons and two neutrons. This is a very stable nucleus as it is doubly magic. The daughter nucleus has two protons and four nucleons fewer than the parent nucleus.



6.1 Kinematics

The “Q-value” of the decay, Q_α is the difference of the mass of the parent and the combined mass of the daughter and the α -particle, multiplied by c^2 .

$$Q_\alpha = (m_P - m_D - m_\alpha)c^2.$$

The mass difference between the parent and daughter nucleus can usually be estimated quite well from the Liquid Drop Model. It is also equal to the difference between the sum of the binding energies of the daughter and the α -particles and that of the parent nucleus.

The α -particle emerges with a kinetic energy T_α , which is slightly below the value of Q_α . This is because if the parent nucleus is at rest before decay there must be some recoil of the daughter nucleus in order to conserve momentum. The daughter nucleus therefore has kinetic energy T_D such that

$$Q_\alpha = T_\alpha + T_D$$

The momenta of the α -particle and daughter nucleus are respectively

$$p_\alpha = \sqrt{2m_\alpha T_\alpha},$$

$$p_D = -\sqrt{2m_D T_D},$$

where m_D is the mass of the daughter nucleus (we have taken the momentum of the α -particle to be positive). Conserving momentum implies $p_\alpha + p_D = 0$ which leads to

$$T_D = \frac{m_\alpha}{m_D} T_\alpha,$$

and neglecting the binding energies, we have

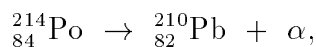
$$\frac{m_\alpha}{m_D} = \frac{4}{A},$$

where A is the atomic mass number of the daughter nucleus. We therefore have for the kinetic energy of the α -particle

$$T_\alpha = \frac{A}{(A+4)} Q_\alpha.$$

Example:

The binding energy of ${}_{84}^{214}\text{Po}$ is 1.66601 GeV, the binding energy of ${}_{82}^{210}\text{Pb}$ (lead) is 1.64555 GeV and the binding energy of ${}_{2}^4\text{He}$ is 28.296 MeV. The Q-value for the decay



is therefore

$$Q_{\alpha} = 1645.55 + 28.296 - 1666.02 = 7.83 \text{ MeV}.$$

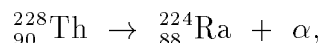
The kinetic energy of the α -particle is then given by

$$T_{\alpha} = \frac{210}{214} \times 7.83 = 7.68 \text{ MeV}.$$

Sometimes the α -particles emerge with kinetic energies which are somewhat lower than this prediction. Such α -decays are accompanied by the emission of γ -rays. What is happening is that the daughter nucleus is being produced in one of its excited states, so that there is less energy available for the α -particle (or the recoil of the daughter nucleus).

Example:

The binding energy of ${}_{90}^{228}\text{Th}$ (thorium) is 1.743077 GeV, the binding energy of ${}_{88}^{224}\text{Ra}$ (radium) is 1.720301 GeV and the binding energy of ${}_{2}^4\text{He}$ is 28.296 MeV. The Q-value for the decay



is therefore

$$Q_{\alpha} = 1720.301 + 28.296 - 1743.077 = 5.52 \text{ MeV}.$$

The kinetic energy of the α -particle is then given by

$$T_{\alpha} = \frac{224}{228} \times 5.52 = 5.42 \text{ MeV}.$$

α -particles are observed with this kinetic energy, but also with kinetic energies 5.34, 5.21, 5.17 and 5.14 MeV.

From this we can conclude that there are excited states of ${}_{88}^{224}\text{Ra}$ with energies of 0.08, 0.21, 0.25 and 0.28 MeV. The α -decay is therefore accompanied by γ -rays (photons) with energies equal to the differences of these energies.

It is sometimes possible to find an α -particle whose energy is *larger* than that predicted from the Q-value. This occurs when the parent nucleus is itself a product of a decay from a further ('grand'-)parent. In this case the parent α -decaying nucleus can be produced in one of its excited states. In most cases this state will decay to the ground state by emitting γ -rays before the α -decay takes places. But in some cases where the excited state is relatively long-lived and the decay constant for the α -decay is large the excited state can α -decay directly and the Q-value for such a decay is larger than for decay from the ground state by an amount equal to the excitation energy.

In the above example of α -decay from ${}_{84}^{214}\text{Po}$ (polonium) the parent nucleus is actually unstable and is produced by β -decay of ${}_{83}^{214}\text{Bi}$ (bismuth). ${}_{84}^{214}\text{Po}$ has excited states with energies 0.61, 1.41, 1.54, 1.66 MeV above the ground state. Therefore as well as an α -decay with Q-value 7.83 MeV, calculated above, there are α -decays with Q-values of 8.44, 9.24, 9.37 and 9.49 MeV.

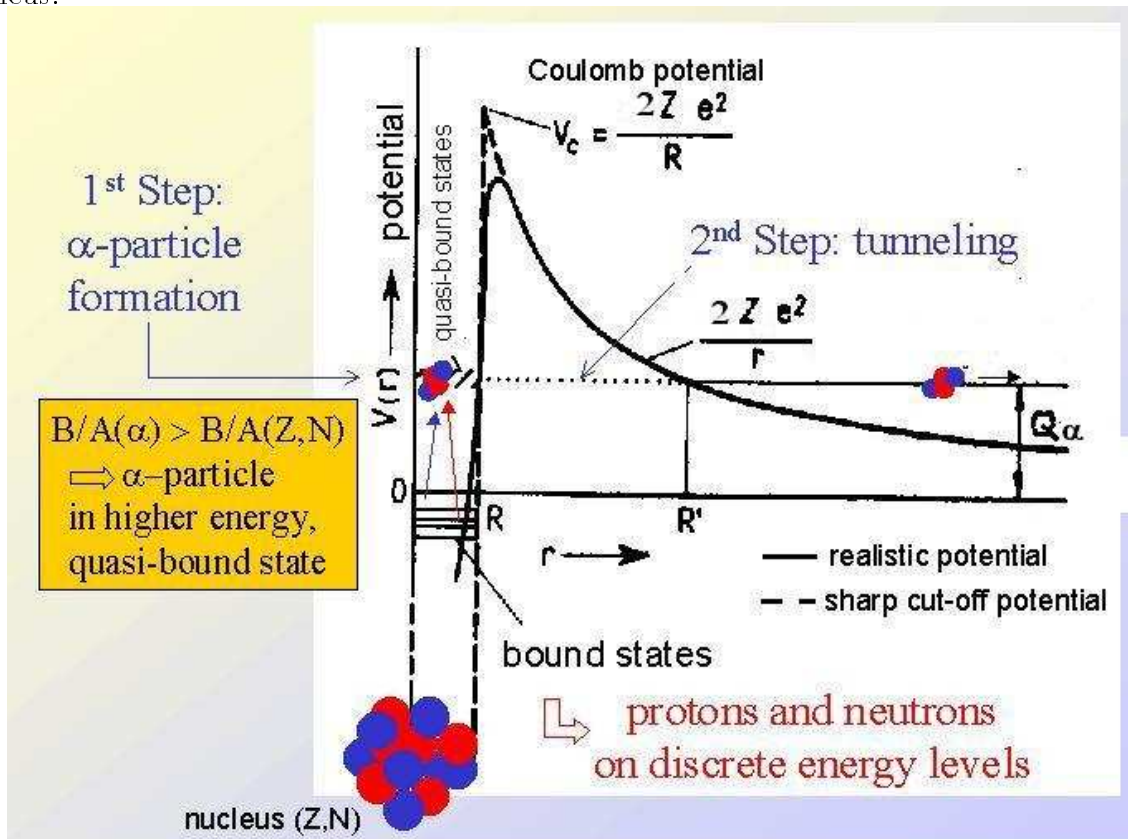
6.2 Decay Mechanism

The mean lifetime of α -decaying nuclei varies from the order of 10^{-7} secs to 10^{10} years.

We can understand this by investigating the mechanism for α -decay.

What happens is that two protons from the highest proton energy levels and two neutrons from the highest neutron energy levels combine to form an α -particle inside the nucleus - this is known as a “quasi-bound-state”. It acquires an energy which is approximately equal to Q_α (we henceforth neglect the small correction due to the recoil of the nucleus).

The α -particle is bound to the potential well provided by the strong, short-range, nuclear forces. There is also a Coulomb repulsion between this ‘quasi-’ α -particle and the rest of the nucleus.



Together these form a potential barrier, whose height, U_0 , is the value of the Coulomb potential at the radius, R , of the nucleus (where the strong interactions are rapidly attenuated).

$$U_0 = \frac{2Ze^2}{4\pi\epsilon_0 R},$$

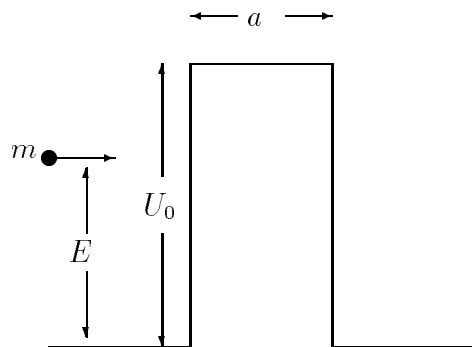
where Ze is the electric charge of the daughter nucleus.

The barrier extends from $r = R$, the nuclear radius to $r = R'$, where

$$Q_\alpha = \frac{2Ze^2}{4\pi\epsilon_0 R'}.$$

Beyond R' the α -particle has enough energy to escape.

Using classical mechanics, the α -particle does not have enough energy to cross this barrier, but it *can* penetrate through via quantum tunnelling.



For a square potential of height U_0 and width a , the tunnelling probability for a particle with mass, m and energy E , is approximately given by

$$T = \exp\left(-2\sqrt{2m(U_0 - E)}\frac{a}{\hbar}\right).$$

It is this exponential which varies very rapidly with its argument, that is responsible for the huge variation in α -decay constants.

This formula applies to a potential barrier of constant height U_0 , whereas for α -decay the potential inside the barrier is

$$U(r) = \frac{2Ze^2}{4\pi\epsilon_0 r}.$$

The result of this is that the exponent in the above expression is replaced by the integral

$$-\frac{2}{\hbar} \int_R^{R'} \sqrt{2m_\alpha \left(\frac{2Ze^2}{4\pi\epsilon_0 r} - Q_\alpha \right)} dr$$

Finally we need to multiply the transition probability by the number of times per sec that the α -particle ‘tries’ to escape, which is how often it can travel from the centre to the edge of the nucleus and back. This is approximately given by

$$\frac{v}{2R},$$

where $v = \sqrt{2Q_\alpha/m_\alpha}$, is the velocity of the α -particle inside the nucleus.

When all this is done we arrive at the approximate result

$$\ln \lambda = f - g \frac{Z}{\sqrt{Q_\alpha}},$$

where

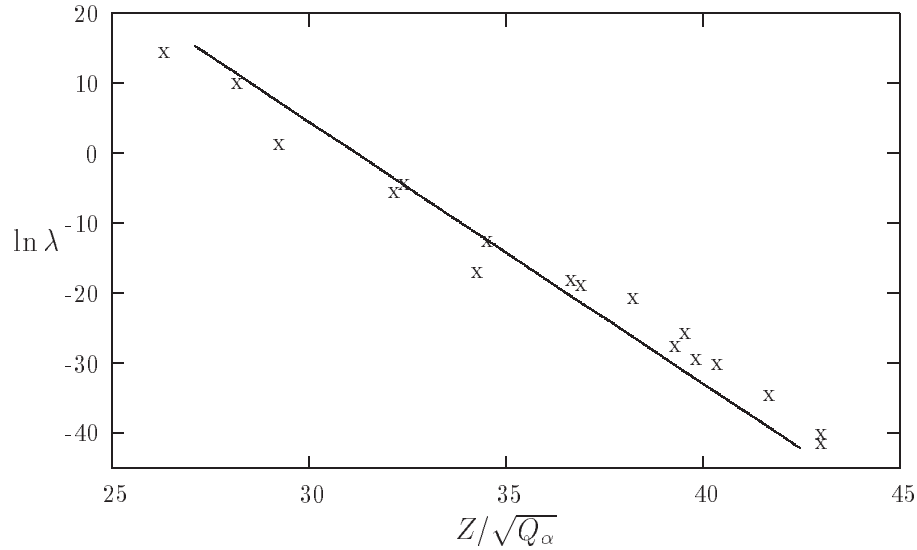
$$g = 2\sqrt{2}\pi\alpha\sqrt{m_\alpha c^2} = 3.97 \text{ MeV}^{1/2},$$

and

$$f = \ln\left(\frac{v}{2R}\right) + 8\sqrt{RZ\alpha m_\alpha c/\hbar}.$$

f varies somewhat for different nuclei but is approximately equal to 128.

This very crude approximation agrees reasonably well with data



We see that as the quantity $Z/\sqrt{Q_\alpha}$ varies over the range 25 - 45, the logarithm of the decay constant varies over a similar range from -45 to 15, but this implies a range of lifetimes from e^{-15} to e^{45} secs (less than a microsecond to longer than the age of the Universe)