7 Beta Decay

β-decay is the radioactive decay of a nuclide in which an electron or a positron is emitted.

$$ {}^{A}_{Z}\{P\} \;\rightarrow\; {}^{A}_{(Z+1)}\{D\} \,+\, e^{-} \,+ \bar{\nu}, $$

or

$$ {}^{A}_{Z}\{P\} \;\rightarrow\; {}^{A}_{(Z-1)}\{D\} \,+\, e^{+} \,+ \nu. $$

The atomic mass number is unchanged so that these reactions occur between "isobars".

The electron (or positron) does not exist inside the nucleus but is created in the reaction

$$ n \;\rightarrow\; p +\, e^{-} \,+ \bar{\nu}. $$

In fact the neutron has a mass that exceeds the sum of the masses of the proton plus the electron so that a free neutron can undergo this decay with a lifetime of about 11 minutes.

Inside a nucleus such a decay is not always energetically allowed because of the difference in the binding energies of the parent and daughter nuclei. When a neutron is converted into a proton the Coulomb repulsion between the nucleons increases - thereby decreasing the binding energy. Moreover there is a pairing term in the semi-empirical mass formula that favours even numbers of protons and neutrons and a symmetry term that tells us that the number of protons and neutrons should be roughly equal.

β-decay is energetically permitted provided the mass of the parent exceeds the mass of the daughter plus the mass of an electron.

$$ M(Z,A) \;>\; M((Z+1),A) \,+\, m_e, $$

for electron emission, and

$$ M(Z,A) \;>\; M((Z-1),A) \,+\, m_e, $$

for positron emission. In the latter case a proton is converted into a more massive neutron, but the binding energy of the daughter may be such that the total nuclear mass of the daughter is less than that of the parent by more than the electron mass, m_e.

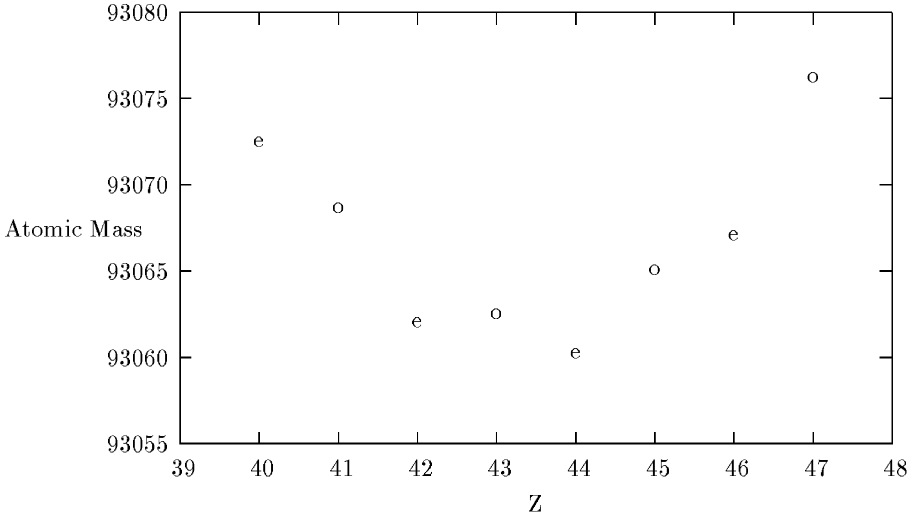
The mass of the electron can be included directly by comparing atomic masses, since a neutral atom always has Z electrons. Thus we require

$$ \mathcal{M}(Z,A) \;>\; \mathcal{M}((Z+1),A) $$

for electron emission. The atomic (as opposed to nuclear) mass included the masses of the electrons. However, this will not work for positron emission, for which Z *decreases* by one unit.

For nuclei with even A, it turns out that because of the pairing term in the binding energy, nuclides with odd numbers of protons and neutrons (odd-odd nuclides) are nearly always unstable against β- decay. On the other hand, even-even nuclides can also sometimes be unstable against β-decay if the number of neutrons in a particular isobar is too large or too small for stability.

For example, consider the isobars for A=100.



We note that all the odd-odd nuclides marked "o" have a larger atomic mass than one of the adjacent even-even (marked "e") nuclides and that for the case of Z=43, *both* electron and positron emission are energetically allowed so that this nuclide (Tc) can decay either by electron emission to Z=44 (Ru) or by positron emission to Z=42 (Mo). Moreover, the even-even Z=40 nuclide (Zr) can decay by electron emission to Z=41 (Nb).

For nuclei with odd A there is either an even number of neutrons *or* an even number of protons. In this case the pairing term does not change from isobar to isobar and the question of stability relies on the balance between the symmetry term which prefers equal numbers of protons and neutrons and the Coulomb terms which prefers fewer protons. For such nuclides there is only one stable isobar, with some atomic number Z_A. This means that the isobars with atomic number $Z > Z_A$ have too many protons for stability can always β-decay emitting a positron, whereas isobars with $Z < Z_A$ have too many neutrons, and can undergo β-decay emitting an electron. The value of Z_A for a given A can be obtained by minimizing the *atomic* mass (including the masses of the electrons) from the semi-empirical mass formula. This gives

$$Z_A \;=\; A\frac{(2a_A + m_n - m_p - m_e)}{(4a_A + a_C A^{2/3})},$$

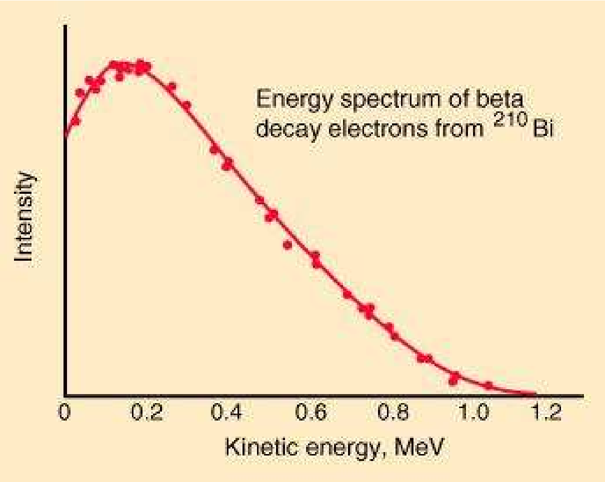
where a_A and a_C are the coefficients of the asymmetry term and Coulomb term in the semi-empirical mass formula.

7.1 Neutrinos

As in the case of α-decay the difference between the mass of the parent nucleus, m_P and the mass of the daughter, m_D plus the electron is the Q-value for the decay, Q_β,

$$Q_\beta \;=\; (m_P - m_D - m_e)c^2,$$

and in this case the recoil of the daughter can be neglected because the electron is so much lighter than the nuclei. We would expect this Q-value to be equal to the kinetic energy of the emitted electron, but what is observed is a spectrum of electron energies up to a maximum value which is equal to this Q-value. For example the intensity of electrons with different energies form the β-decay of ${}^{210}_{83}$Bi (bismuth) is



There is a further puzzle. Since the number of spin-$\frac{1}{2}$ nucleons is the same in the parent and daughter nuclei, the difference in the spins of the parent and daughter nuclei must be an integer. But the electron also has spin-$\frac{1}{2}$, so there appears to be a violation of conservation of angular momentum here.

The solution to both of these puzzles was provided in 1930 by Pauli who postulated the existence of a massless neutral particle with spin-$\frac{1}{2}$ which always accompanies the electron in β-decay. This was called a neutrino. Neutrinos interact very weakly with matter and so they were not actually detected until 1953 (by Reines and Cowan). The fact that the neutrino has spin-$\frac{1}{2}$ means that the total angular momentum can be conserved (if necessary the electron-antineutrino system has orbital angular momentum) and the Q-value is the sum of the energies of the electron and antineutrino. The kinetic energy of the electron can vary from zero (strictly arbitrarily small) where all the Q-value is taken by the antineutrino (the momentum being conserved by the small recoil of the daughter nucleus) to the Q-value in which case the energy carried off by the antineutrino is negligible.

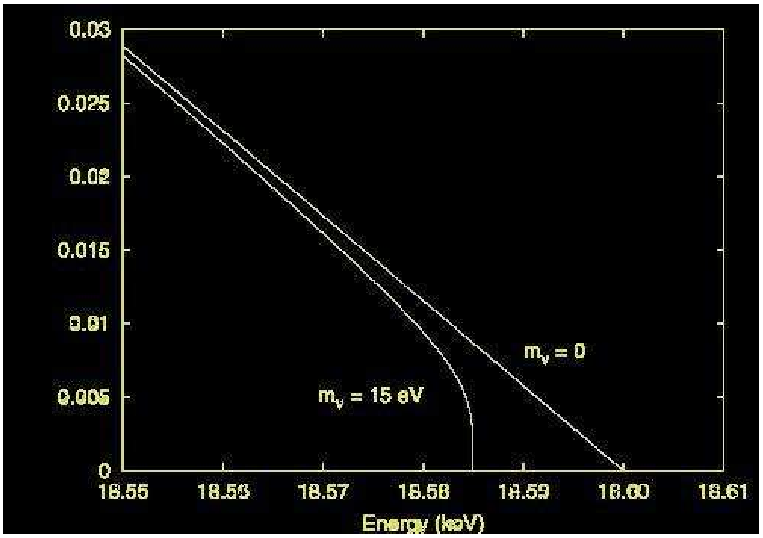
Electrons and neutrinos are examples of "leptons" which are particles that do not interact under the strong nuclear forces - they are not found inside nuclei.

By convention, electrons and neutrinos are assigned a "lepton number" of 1, which means that positrons and antineutrinos have a lepton number of -1. Lepton number is conserved so that it is actually an antineutrino that is emitted together with electron emission β-decay and a neutrino together with positron emission.

The fact that the neutrino has (almost) zero mass is deduced by examining the end-point of the electron energy spectrum. For example for the decay

$$ {}^{3}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + e^{-} + \bar{\nu}, $$

with a Q-value of 18.6 KeV,



For a massless neutrino its total (relativistic) energy can be arbitrarily small and the electron can carry energy up to the Q-value. If the neutrino has a mass, m_ν then the minimum energy that it can have is $m_\nu c^2$, and the electron energy spectrum drops off sharply at the end-point.

It is now known that neutrinos *do* have a tiny mass. The first hint of this was during the observation of the Supernova in 1987, when a burst of neutrinos were observed a few seconds after the burst of γ-rays, implying that the neutrinos had not travelled form the Supernova with exactly the speed of light. This was confirmed by neutrino observation experiments at the Kamiokande neutrino detector in Japan in 1999. However the mass of the neutrino is almost certainly smaller than 0.1 eV/c^2 (compared with the electron mass of 0.511 MeV/c^2). For our purposes we may neglect the neutrino mass.

7.2 Electron Capture

Nuclei which can β-decay emitting a positron and an neutrino, can also decay by another mechanism.

$$e^- + {}^A_Z\{P\} \rightarrow {}^{A}_{(Z-1)}\{D\} + \nu.$$

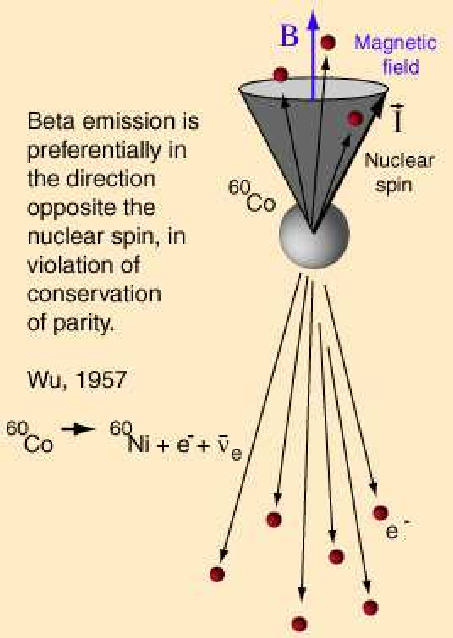
What happens here is that an atom can absorb an electron from one of the inner shells (usually the innermost shell, which is called the "K-shell") and be converted into an atom with one lower atomic number. The energy is entirely carried away by the neutrino and is nearly always undetected because neutrinos interact so weakly with matter.

7.3 Parity Violation

β-decay exhibits a further peculiarity. This was discovered in 1957 by C.S. Wu who observed the decay of radioactive cobalt into nickel

$${}^{60}_{27}\mathrm{Co} \rightarrow {}^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}.$$

The cobalt sample was kept a low temperature and placed in a magnetic field so that the spin of the cobalt was pointing in the direction of the magnetic field.



She discovered that most of the electrons emerged in the opposite direction from the applied magnetic field. If we write $\mathbf{s}$ for the spin of the parent nucleus and $\mathbf{p_e}$ for the momentum of an emitted electron, this means that the average value of the scalar product $\mathbf{s}\cdot\mathbf{p_e}$ was negative. In order to balance the momentum the antineutrinos are usually emitted in the direction of the magnetic field, so that the average value of $\mathbf{s}\cdot\mathbf{p}_{\bar{\nu}}$ was positive.

Under the parity operation

$$\mathbf{r} \;\rightarrow\; -\mathbf{r}$$

and

$$\mathbf{p} \;\rightarrow\; -\mathbf{p}$$

but angular momentum which is defined as a vector product

$$\mathbf{L} \;=\; \mathbf{r}\times\mathbf{p},$$

is unchanged under parity

$$\mathbf{L} \;\rightarrow\; \mathbf{L}.$$

Spin is an internal angular momentum and so it also is unchanged under parity.

But this means that the scalar product $\mathbf{s}\cdot\mathbf{p_e}$ does change under parity

$$\mathbf{s}\cdot\mathbf{p_e} \;\rightarrow\; -\mathbf{s}\cdot\mathbf{p_e}$$

so that the fact that this quantity has a non-zero average value (or expectation value in quantum mechanics terms) means that the mechanism of β-decay violates parity conservation

If we viewed the above diagram in the corner of a mirrored room so that all the directions were reversed the spin would point in the same direction, but the electron direction would be reversed so that in that world the electrons would prefer to emerge in the direction of the magnetic field.

The spin of the daughter nucleus ${}^{60}_{28}\mathrm{Ni}$ is 4 (it is produced in an excited state) whereas that of the parent ${}^{60}_{27}\mathrm{Co}$ was 5, so that in order to compensate for unit of angular momentum lost (in the direction of the magnetic field) the angular momentum the antineutrinos and electrons have their spins in the direction of the magnetic field. This means that the antineutrinos have a spin component $+\frac{1}{2}$ in their direction of motion (in units of $\hbar$) whereas the electrons have a spin component $-\frac{1}{2}$ in their direction of motion. The sign of the component of the spin of a particle in its direction of motion is called the "helicity" of the particle. Neutrinos always have negative helicity (antineutrinos always have positive helicity). An electron can have component of spin either $+\frac{1}{2}$ or $-\frac{1}{2}$ in its direction of motion (either positive or negative helicity). However, the electrons emitted in β-decay usually have negative helicity (positrons emitted in β-decay usually have positive helicity). This means that the mechanism responsible for β-decay (called the "weak interaction") distinguish between positive and negative helicity and therefore violate parity.