

## 8 Gamma Decay

As we have seen  $\gamma$ -decay is often observed in conjunction with  $\alpha$ - or  $\beta$ -decay when the daughter nucleus is formed in an excited state and then makes one or more transitions to its ground state, emitting a photon whose energy is equal to the energy difference between the initial and final nuclear state. These energy differences are usually of order 100 KeV so the photon is well in the  $\gamma$ -ray region of the electromagnetic spectrum.

The lifetime of excited nuclear states is usually of the order of  $10^{-13} - 10^{-12}$  s., so the lifetime is far too short to be measured.

The decay rate (inverse of the mean lifetime) depends on the energy of the photon emitted and the ‘type’ of radiation.

As in the case of Atomic Physics the transition amplitude is proportional to the matrix element of the electric field between the initial and final wavefunctions of the nucleon that makes the transition. This electric field has a space dependence that may be written

$$E = E_0 e^{i\mathbf{k}\cdot\mathbf{r}},$$

where  $\mathbf{k}$  is the wavevector of the emitted photon. For photons of energy 100 KeV and a nucleus of radius a few fm,  $\mathbf{k}\cdot\mathbf{r}$  is much less than 1 and it is sufficient to expand this exponential to first order.

The transition amplitude is therefore proportional to

$$A \propto \int \Psi_f^*(\mathbf{r}) \mathbf{k}\cdot\mathbf{r} \Psi_i(\mathbf{r}) d^3\mathbf{r},$$

where  $\Psi_i$  and  $\Psi_f$  are the initial and final wavefunctions of the proton that makes the transition. This is called “electric dipole” transition (there is no “electric monopole” transition from the first term in the expansion of the exponential because  $\Psi_f(\mathbf{r})$  and  $\Psi_i(\mathbf{r})$  are orthogonal wavefunctions).

The rate for such transition is well approximated by the formula

$$\lambda = 10^5 E_\gamma^3 A^{2/3},$$

where  $E_\gamma$  is the energy of the photon in KeV. The factor of  $A^{2/3}$  is understood from the fact that the transition amplitude is proportional to the nuclear radius, which is in turn proportional to  $A^{1/3}$  (the transition rate is proportional to the square of the amplitude). For photons with energy of order 100 KeV and A of order 100 this gives  $2.5 \times 10^{12} s^{-1}$ .

However, for the above electric dipole matrix element to be non-zero we require certain conditions on the spin and parity of the initial and final states. As in Atomic Physics, the photon carries away one unit of angular momentum, so that the initial and final nuclear spins have to obey the selection rule

$$\Delta I = 0, \pm 1 \quad (I = 0 \rightarrow I = 0 \text{ forbidden})$$

Furthermore since  $\mathbf{r}$  is odd under parity reversal, we require the initial and final states to be of opposite parity, which means that the orbital angular momentum changes by one unit.

If the parity of the initial and final states are the same then the transition is still allowed, but this means that the photon carries away the angular momentum by flipping the spin of the nucleon that makes the transition. For this to happen the magnetic moment of the nucleon interacts with the magnetic field component of the electromagnetic wave associated with the emitted photon. This is called a “magnetic dipole transition” amplitude, and for such a process the transition amplitude is suppressed relative to the amplitude for a typical electric dipole transition by about a factor of

$$\frac{\hbar c}{m_p R}$$

which is about 0.1 for a nucleus of radius a few fm. (and therefore .01 suppression of the decay rate).

Transitions between nuclear states in which the photon is required to carry off more than one unit of angular momentum are permitted. This is because the photon can acquire orbital angular momentum relative to the recoiling nucleus. Thus the total angular momentum change,  $L$  in a nuclear transition can take the values

$$|I_i - I_f| \leq L \leq |I_i + I_f|,$$

where  $I_i$  and  $I_f$  are the initial and final nuclear spins. However there is a price to pay in terms of transition rates. For each increase in  $L$  there is a suppression in the transition amplitude of  $kR$ , because these higher multipole transitions arise from higher orders in the expansion of  $\exp(i\mathbf{k} \cdot \mathbf{r})$ . For a nucleus of radius a few fm and a photon energy of 100 KeV is a factor of  $10^{-3}$  ( so a factor of  $10^{-6}$  in the rate). There is a further suppression factor for higher values of  $L$ . A transition will proceed by the lowest allowed value of  $L$ .

This is also subject to selection rules for the parity difference between initial and final states, namely

$$\Delta P = (-1)^L,$$

for electric transitions (written E{L}) with angular momentum  $L$ , and and

$$\Delta P = (-1)^{L-1},$$

for the (even further suppressed) magnetic transitions.

Thus from the initial and final nuclear spins and parities we can determine the “multipolarity” of the transition and whether it is electric or magnetic.

Here are some examples

$$\begin{aligned} 2^+ &\rightarrow 1^-, & E1, \\ 2^+ &\rightarrow 1^+, & M1, \\ 3^+ &\rightarrow 1^-, & M2, \end{aligned}$$

$$3^+ \rightarrow 1^+, \quad E2.$$

Most electromagnetic transitions from an excited state to the ground state have a lifetime which is too short to be measured (less than  $1 \mu\text{s}$ ). However, in the Shell Model the energy levels sometimes arrange themselves such that there is a very high spin excited state next to a low spin ground state or vice versa. Such a transition is only permitted by a high multipolarity transition and therefore proceeds very slowly. The excited states then live long enough for their lifetime to be measured and can even be as long as several years. An example is the nuclide  ${}_{56}^{137}\text{Ba}$  (barium) which has an excited state with spin and parity  $\frac{11}{2}^-$  next to a ground state of  $\frac{3}{2}^+$ . The transition is M4 and the excited state has a mean lifetime of around 200 s. These metastable excited states are called “isomers” and there are regions of the Periodic Table known as “islands of isomers” where such metastable excited states are quite common.

## 8.1 The Mössbauer Effect

In Atomic Physics, it is possible to excite atoms into their excited states by bombarding them with photons with the resonant frequencies, i.e. with energies equal to the energies between the ground state and the excited states.

In nuclei this is not usually possible. The reason for this is to with the small nuclear recoil. The energy of the emitted photon,  $E_\gamma$  is not exactly equal to the excitation energy  $E_0$ . The photon carries momentum  $E_\gamma/c$  and so the recoiling nucleus must have equal and opposite momentum. Consequently it acquires a recoil kinetic energy of

$$T = \frac{E_\gamma^2}{2M_N c^2},$$

where  $M_N$  is the nuclear mass. The de-excitation energy  $E_0$  is the sum of the photon energy plus this kinetic energy

$$E_0 = E_\gamma + \frac{E_\gamma^2}{2M_N c^2},$$

which has approximate solution

$$E_\gamma = E_0 \left( 1 - \frac{E_0}{2M_N c^2} \right)$$

For a photon of energy 100 KeV and a nucleus with  $A=100$ , this recoil energy is about 0.05 eV. Furthermore if we now use the emitted photon to bombard a similar nuclide with the hope of exciting it, we find that the target nucleus also recoils so that the energy that it can absorb in its own rest frame,  $E'_0$  is given by

$$E'_0 = E_\gamma \left( 1 - \frac{E_0}{2M_N c^2} \right) \approx E_0 \left( 1 - \frac{E_0}{M_N c^2} \right),$$

so that  $E'_0$  falls short of  $E_0$  by about 0.1 eV (in the above example).

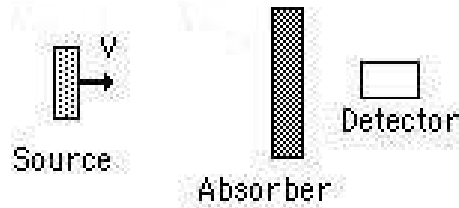
This may not seem much for a photon of energy 100 KeV, but the problem is that even for fast decaying excited states with lifetimes,  $\tau$ , of about  $10^{-12}$  s., the line-width is given by

$$\Gamma = \frac{\hbar}{\tau} \approx 10^{-3} \text{ eV},$$

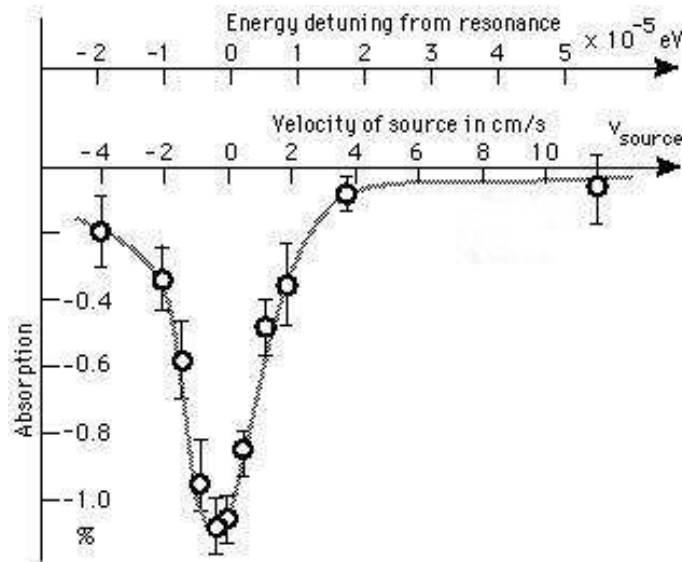
so the difference between the excitation energy  $E_0$  and the energy  $E_0$  that the recoiling nucleus can absorb is much larger than the width of the photon, thereby making the absorption impossible.

The way out of this was discovered by Mössbauer. If the source and target nuclei are both fixed in a crystal lattice then the recoil momentum can be taken up by the entire crystal (whose mass is many orders of magnitude larger than that of the nucleus) and the recoil energy is negligible.

This is called the Mössbauer effect and it provides an extremely accurate method for measuring the widths of nuclear transitions.



The source and target are both fixed to a crystal and if the source is stationary the intensity of  $\gamma$ -rays reaching the detector is small because most of them are absorbed by the target.



If the source is moving by as little as a few cm per second there is an increase in the intensity of  $\gamma$ -rays reaching the detector, because the Doppler effect of the  $\gamma$ -rays from the source causes the incident photons to be just off-resonance. Line widths can be measured this way to an accuracy of  $10^{-5}$  eV. If the source is moved with velocity  $v$  then using the Doppler shift the difference  $\Delta\lambda$  between the wavelength of the emitted photon (in the rest

frame of the emitter) and the wavelength of the absorbed photon is

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c},$$

(we are able to use the non-relativistic Doppler effect for such small velocities). In terms of photon energies we may write this as

$$\frac{\Delta E}{E} = \frac{v}{c}.$$

Now if for this velocity the absorption has fallen to approximately one half of the peak absorption (for  $v = 0$ ) then this value of  $\Delta E$  corresponds to the half width,  $\frac{1}{2}\Gamma$  of the spectral line. Thus we end up with an expression for the line-width for a photon of energy  $E$

$$\Gamma = 2E\frac{v_{1/2}}{c},$$

where  $v_{1/2}$  is the velocity for which the absorption falls to one-half of its peak value.