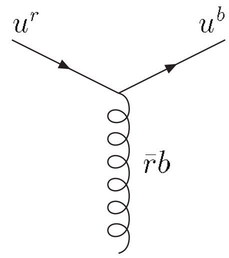
18 Quantum Chromodynamics (QCD)

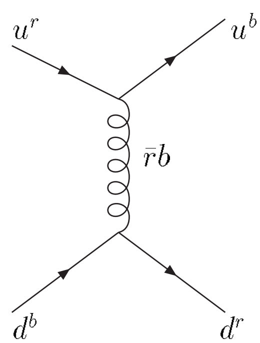
18.1 Gluons and Colour

In the same way that in weak interactions, the weak gauge bosons $W^{\pm}$ can effect changes of flavour when they interact with quarks, in the case of strong interactions the strong gauge bosons (gluons) can effect changes of colour of the quarks (but conserve flavour). Thus we get interaction vertices of the form



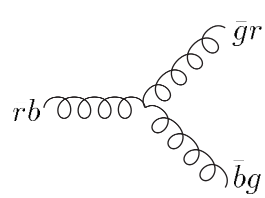
which converts a red quark into a blue quark. The fact that the flavour is unchanged is the reason why flavour is conserved in strong interactions. There are 6 such colour changing gluons and in addition two colour neutral gluons (like the Z in weak interactions) which do not change colour, making a total of 8 gluons (the reason that there are 8 gluons comes from a group theory analysis of the theory of colour - outside the scope of these lectures),

Strong interaction processes consist of quarks exchanging gluons and (usually) changing colour, e.g



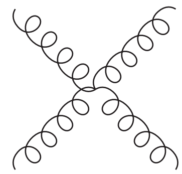
This theory of strong interactions, developed in the 1970's is called Quantum Chromodynamics (QCD).

In the same way that the Z can couple to the $W^{\pm}$, so gluons can couple to each other with vertices such as



(total colour must be conserved)

There are also vertices between four gluons of the form



In the case of weak and electromagnetic interactions, the strengths of the couplings of the gauge bosons to quarks and leptons, controlled by the electron charge, e, and the weak coupling, g_W, are sufficiently small

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{1}{137}, \qquad \alpha_W = \frac{g_w^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{30}$$

so that we can calculate the rates for weak and electromagnetic processes using perturbation theory, with higher order corrections being of order α, for electromagnetic processes, and α_W for weak processes. However, for the strong interactions inside a nucleus, the coupling constant is too large for this to be possible. It is for this reason that we cannot, even in principle, calculate the energy levels of nuclei.

18.2 Running Coupling

However, QCD is not entirely useless.

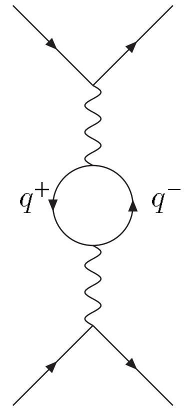
It turns out that at sufficiently high energy/momentum scales, Q, the effective strong coupling becomes small.

It is convenient to work in terms of α_s where

$$\alpha_s = \frac{g_s^2}{4\pi\epsilon_0\hbar c}.$$

where g_s is the coupling of the gluons to quarks or the coupling of gluons to each other.

The reason that this becomes small is 'negative screening'. When an electric charge is probed by another charge, the virtual photon exchanged between them can sometimes create a pair of charged particles (a particle and its antiparticle), which exist for a short while before annihilating each other again. Diagrammatically we would represent this as



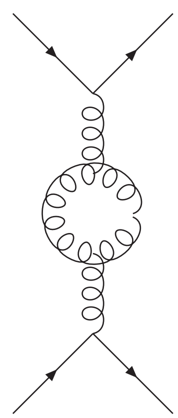
The effect is to surround the probed charge by a cloud of charged particles which act as a screen - reducing the effective measured charge. As the energy/momentum scale increases and the probe penetrates further into this screen the measured charge increase.

When we write

$$\alpha = \frac{1}{137}$$

this refers to low energy/momentum measurements of the electron charge. At a momentum scale $Q \sim M_Z c$ the value is closer to 1/129.

In the case of QCD, we can also have processes in which a cloud of gluons can be produced by the exchanged virtual gluon, because gluons interact with each other (unlike photons). Thus we have diagrams like



The effect of this is negative screening and it turns out that at large momenta the effective coupling decreases.

Mathematically we describe the momentum scale dependence (running) of the coupling in terms of a function known as the β-function, defined as

$$\beta = \frac{d\alpha(Q)}{d\ln Q^2}$$

For electromagnetism, β is positive so that α increases with increasing Q.

But for QCD, we have an expansion of β as a power series in α_s,

$$\beta \;=\; \alpha_s^2\beta_0 \,+\, \mathcal{O}(\alpha_s^3).$$

where

$$\beta_0 \;=\; -\frac{1}{4\pi}\left(11 - \frac{2}{3}n_f\right)$$

Here n_f means the number of active flavours and is used in the same way as in the calculation of R in $e^+e^- \rightarrow$hadrons.
For $Q < 2m_c c,\ n_f = 3,$
for $2m_c c < Q < 2m_b c,\ n_f = 4$
for $Q > 2m_b c,\ n_f = 5.$
In this expression for β the $-11/(4\pi)$ comes from the interaction of the gluons with each other producing a gluon cloud which decreases the effective coupling with increasing Q, whereas the term proportional to n_f (with a positive sign) comes from the creation of quark-antiquark pairs by the virtual gluon and is similar to the producing of charged pairs in electromagnetism - so its effect on the running coupling has the same sign as in electromagnetism.

The solution to this differential equation (neglecting the higher order terms in α_s) is

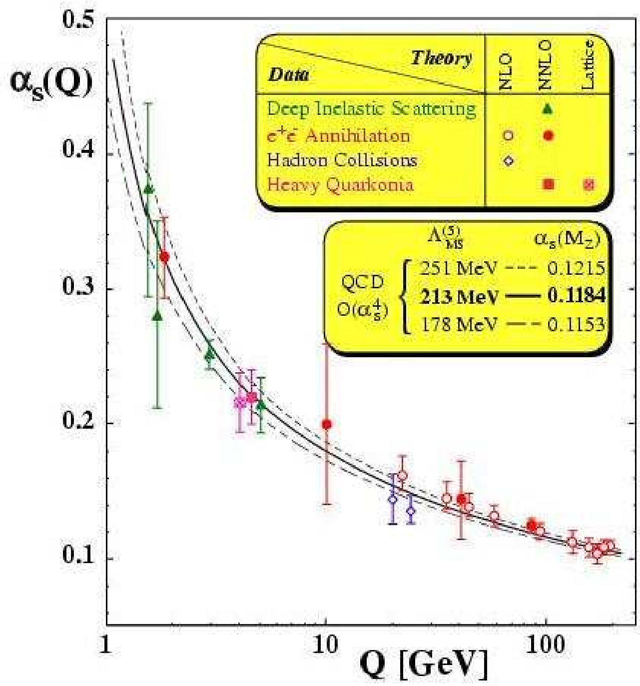
$$\alpha_s(Q) \;=\; \frac{\alpha_s(\mu)}{(1 - \beta_0\alpha_s(\mu)\ln(Q^2/\mu^2))}$$

where $\alpha_s(\mu)$ is the value of α_s at some reference momentum scale (it serves as the integration constant for the differential equation). Usually this is taken to be $\mu = M_Z c$, since the value of α_s was measured very accurately at LEPI at this scale and its value was found to be

$$\alpha_s(M_Z c) \;=\; 0.12,$$

which is not too large.

Experimental measurements of α_s over a large range of energy/momentum scales agrees well with this formula.



We see that for Q greater than a few GeV, $\alpha_s(Q)$ is small enough that we would expect a calculation using perturbation theory to be fairly reliable. Below these energy/momentum scales (e.g. inside the nucleus) we cannot use perturbation theory and QCD is not very helpful.

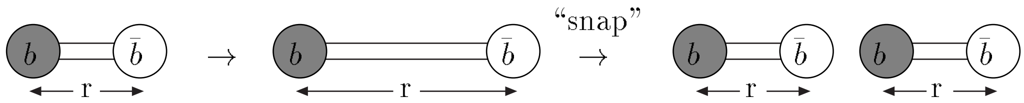
The property of QCD that the effective coupling decreases with increasing energy/momentum is called "asymptotic freedom".

18.3 Quark Confinement

We have seen that the weak interactions are short-range because the gauge bosons $W^{\pm}$ and Z are massive and so the weak potential is of the Yukawa type with an exponential fall-off with distance $\exp(-M_W cr/\hbar)$.

In the case of QCD the gluons are massless, so we might expect the strong interactions to be long-range (as in the case of electromagnetic interactions mediated by massless photons), whereas we know that the strong interactions have a range of a few fermi.

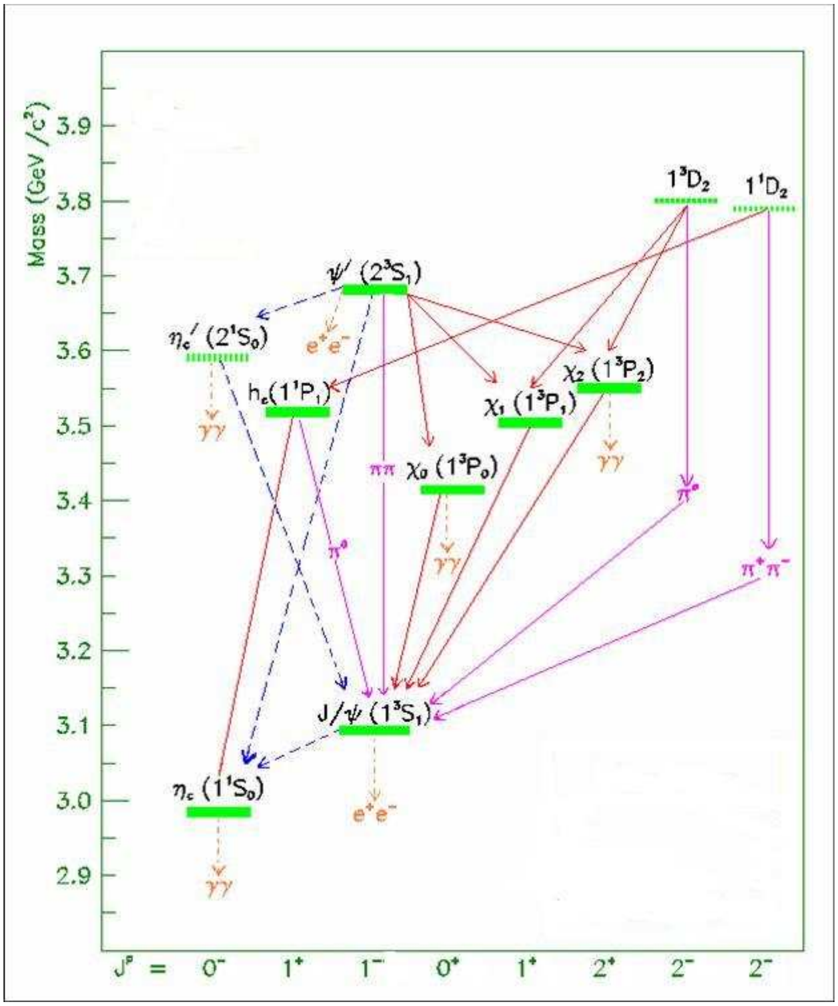
The answer to this puzzle is the converse of asymptotic freedom. At large momentum, where we are probing short distances, the effective coupling decreases. Conversely at large quark separations the effective coupling increases and the binding between them gets stronger.



It is not possible to isolate a single quark or gluon. Consider a meson, which is a quark-antiquark state of the opposite colour (e.g. red and anti-red) bound together by a 'string' of gluons. As we try to pull the quark and antiquark apart, the tension in the string increases and eventually the string will 'snap' producing a quark a the end of the part of the string containing the antiquark (of opposite colour) and likewise and antiquark of opposite colour at the end of the part of the string containing the quark. So we end up with two mesons, both of which are colour singlets (colourless), but we do not succeed in isolating a single quark or antiquark.

The only hadron states that we can observe are colourless (colour singlet) states - either mesons which are superpositions of quark-antiquark pairs of opposite colours, or baryons which consist of three quarks but which are antisymmetric under the interchange of any two quark colours. This is known as "quark confinement". Its exact mechanism is not understood, but numerical studies in QCD confirm that this confinement does indeed take place.

18.4 Quark-antiquark Potential and Heavy Quark Bound States



At momentum scales $Q \sim 2m_c c$, the running coupling is $\alpha_s \sim 0.3$ which is small enough to use perturbation theory to obtain energy levels for the J/Ψ (also known as "charmonium"), by solving the Schrödinger equation using a potential which contains a term that represents the confinement,

$$V(r) \;=\; -\frac{4}{3}\frac{\alpha_s}{r} + kr.$$

The first term is the usual Coulomb-like potential which dominates at short distances where the potential can be viewed as the exchange of massless gluons - the factor of 4/3 is associated with the number of quark colours and the number of gluons. The term kr increases with increasing separation and represents confinement. By fitting the data, k is found to be about 0.85 Gev fm^{-1} (1.3×10^5 N. !).

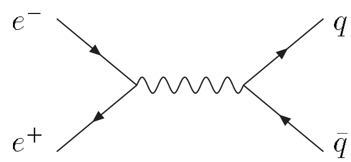
Using this potential and making corrections for relativistic effects and spin-orbit coupling, the spectrum of the $c - \bar{c}$ system (J/Ψ and its excited states) and also the $b - \bar{b}$ system (Υ and its excited states) can be obtained to a high degree of accuracy.

18.5 Three Jets in Electron-positron Annihilation

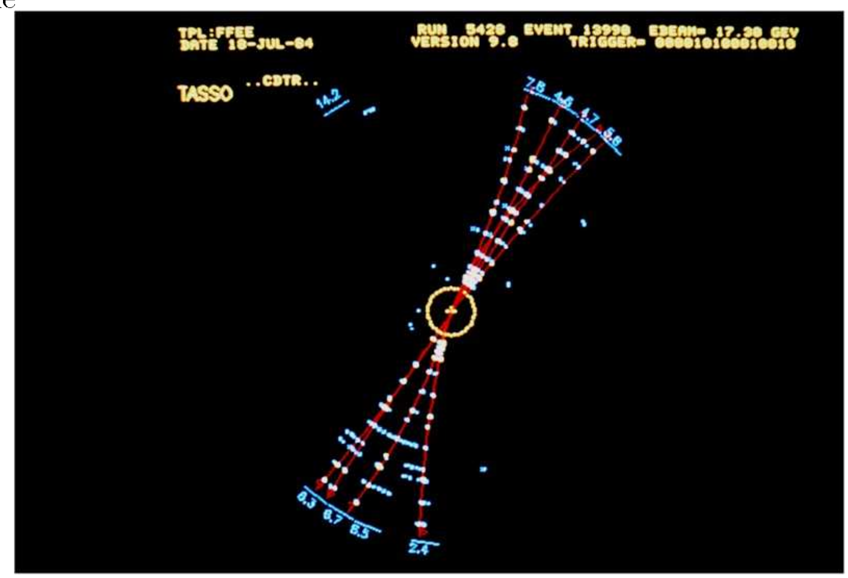
When we were considering the process

$$e^+ + e^- \rightarrow \text{hadrons}$$

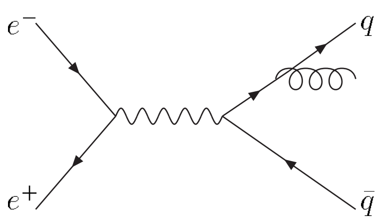
the Feynman diagram considered was

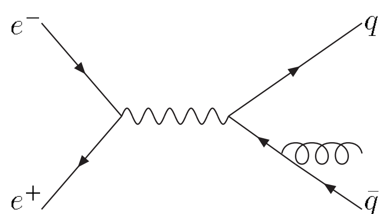


The final state quarks fragment to produce two jets of hadrons moving in opposite directions. Such two-jet events were observed in the $e^+ e^-$ experiment at DESY in the 1970's. A typical event looks like



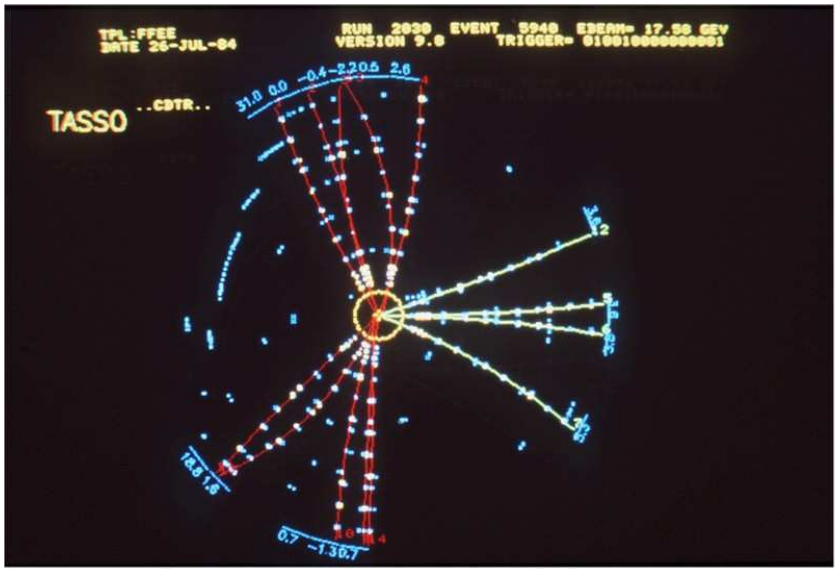
Because quarks interact with gluons one can also have Feynman diagrams





which have a quark, an antiquark, and a gluon in the final state. The gluon also fragments into a hadron jet and so we get three jets of particles.

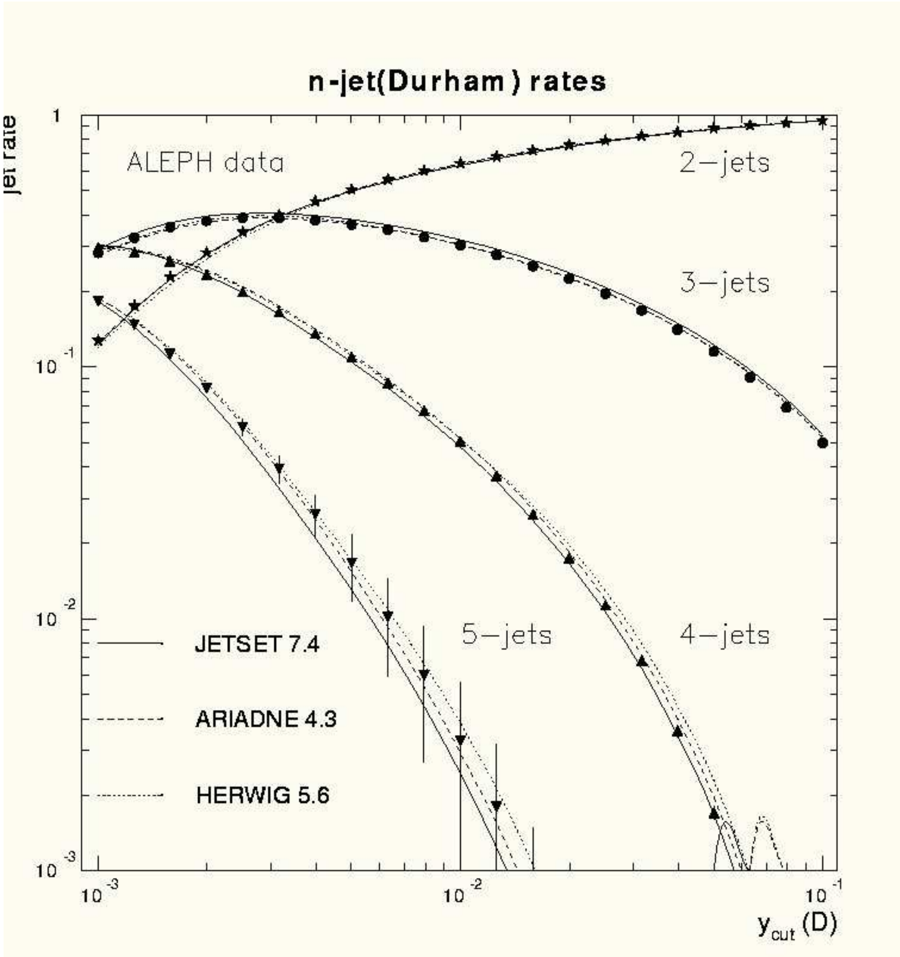
The first such events we observed at DESY in 1979



Since gluons cannot be isolated and observed directly, this was taken as the first piece of evidence that gluons existed and coupled to quarks.

Four and five jet events have now also been observed. At LEP energies (100 - 200 GeV) the running coupling is small ($\alpha_s \sim 0.1$). For this reason three jet events are rarer than two jet events because the amplitude for the process contains a factor of the gluon coupling g_s and so the rate is expected to be suppressed relative to the two jet rate by a factor of order $\alpha_s \propto g_s^2$. Likewise four and five jets events are even rarer

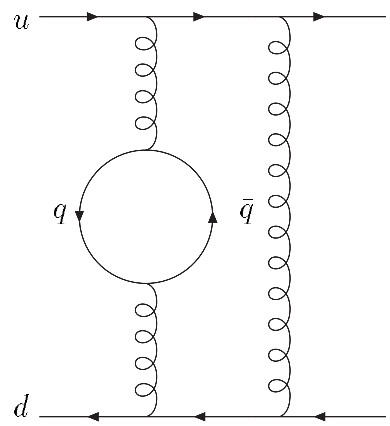
The exact definition of a jet depends on how big an opening angle constitutes a single jet. This is parametrised by a kinematic variable called y_{cut} (this is a measure of the maximum fraction of the total energy that can be contained in a single jet). Because of the small running coupling, perturbative QCD can be used to calculate the number of jets as a function of this y_{cut}



Some correction to the result from pure perturbative QCD has to be made for the process of fragmentation. The different curves shown are for different models used to simulate this fragmentation process. Nevertheless the agreement between the data and QCD theory is impressive.

18.6 Sea Quarks and Gluon content of Hadrons

The quarks (and antiquarks) inside hadrons are bound together by exchanging gluons. Thus, as well as having the quarks inside hadrons there will be gluons. These gluons can in turn create quark-antiquark pairs (which exist for a very short time and then annihilate). Diagrammatically the 'inside' of a π^- may look like



Thus, inside the hadron, we have the main quarks, called valence quarks which determine the quantum numbers (flavour) of the hadron and in addition a cloud of quark-antiquark pairs created by the gluons exchanged between the valence quarks. These extra quark-antiquark pairs are called "sea quarks".

18.7 Parton Distribution Functions

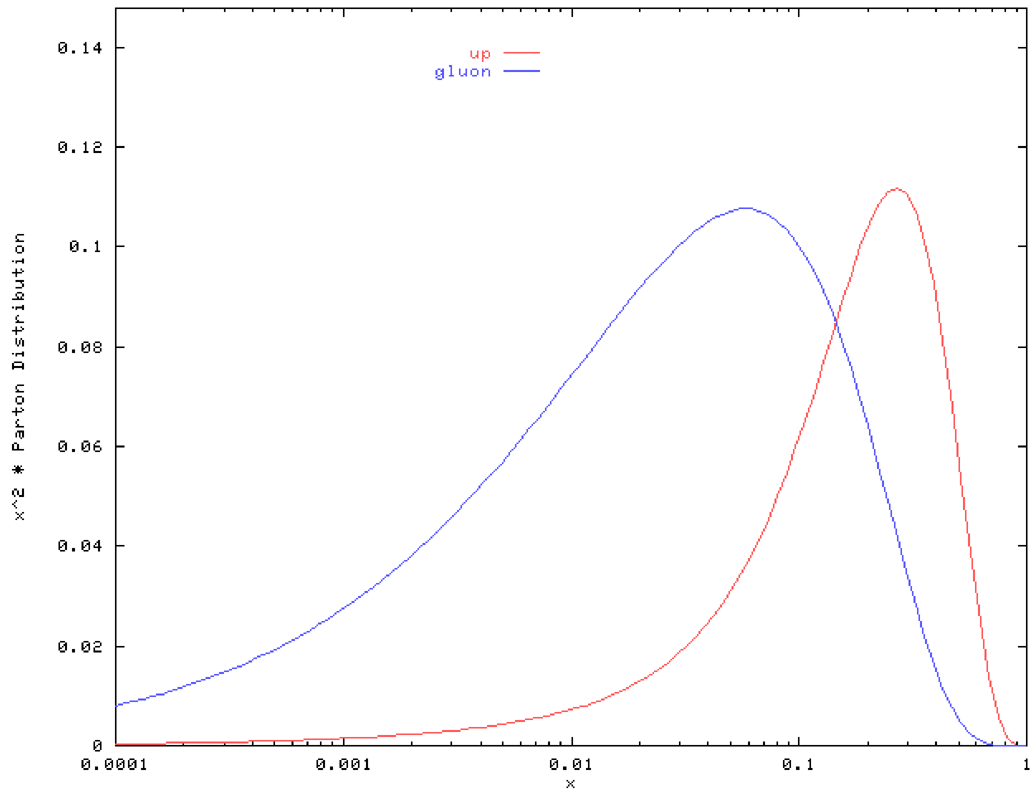
Quarks, antiquarks and gluons are collectively known as "partons". If we consider a relativistically moving hadron ($\approx pc$), some fraction, x (known as "Bjorken-x") will be carried by a parton of each possible type. The probability that a fraction, x, of the momentum of the hadron (say a proton) is carried by a parton of type i is called the "parton distribution function" and is written as

$$f_h^i(x)$$

where i can mean a gluon, quark, or antiquark of any given flavour.

It is not possible to calculate these parton distribution functions in QCD, but they can be inferred by examining experimental data. Once they are known QCD can be used to predict scattering cross-sections for other processes.

An example of these parton distributions (as a function of x) is

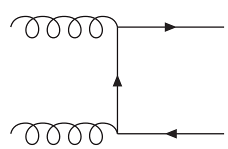


18.8 Factorization

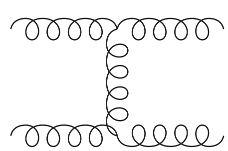
Perturbative QCD can be used to calculate cross-sections at the parton level, provided that the energy/momentum scale of the process, Q is large enough so that $\alpha_S(Q)$ is sufficiently small.

For, example we can calculate the cross-section for the processes

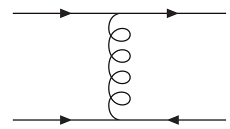
- $$g, \, + g \;\; \rightarrow \;\; q + \bar{q}$$



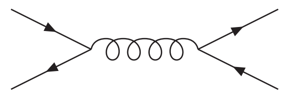
- $$g, \, + g \;\; \rightarrow \;\; g + g$$



- $$q, \, + q \;\; \rightarrow \;\; q + q$$



- $$q + \bar{q} \rightarrow q + \bar{q}$$



etc.

Denote the calculated differential cross-section for two partons of type i and j to go into two other partons with momentum p_T transverse to the direction of the incoming partons by

$$\frac{d\hat{\sigma}(\hat{s})}{dp_T},$$

where $\sqrt{\hat{s}}$ is the centre-of-mass energy of the incoming partons.

What we are really interested in is a process in which the initial states are not partons (which cannot be isolated in a laboratory owing to confinement) but initial state hadrons such as a proton and an antiproton.

In order to obtain the differential cross-section for proton-antiproton scattering into two jets of final state hadrons with transverse momentum, p_T we can invoke the factorisation theorem.

If we pull a parton of type i from one of the incoming protons, with a fraction x_1 of the momentum of the parent proton, and a parton of type j from the antiproton, with a fraction x_2 of its momentum, then (in the case of relativistically moving particles whose energy E and momentum $\mathbf{p}$ are related by $E \approx |\mathbf{p}|c$) the centre-of-mass energy of the two partons is given by

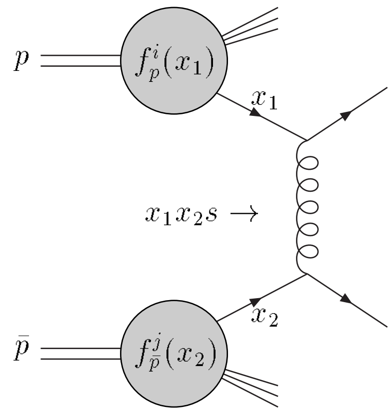
$$\hat{s} = x_1 x_2 s$$

(where $\sqrt{s}$ is the centre-of-mass energy of the incoming proton and antiproton).

Factorisation tells us that if $f_p^i(x_1)$ and $f_{\bar{p}}^j(x_2)$ are the parton distribution functions for partons i and j, then the contribution to the proton-proton differential cross-section due to this particular parton scattering is

$$\int_0^1 dx_1 \int_0^1 dx_2 f_p^i(x_1) f_{\bar{p}}^j(x_2) \frac{d\hat{\sigma}(x_1 x_2 s)}{dp_T}$$

For example if the parton level scattering is quark-quark scattering we can represent this contribution as



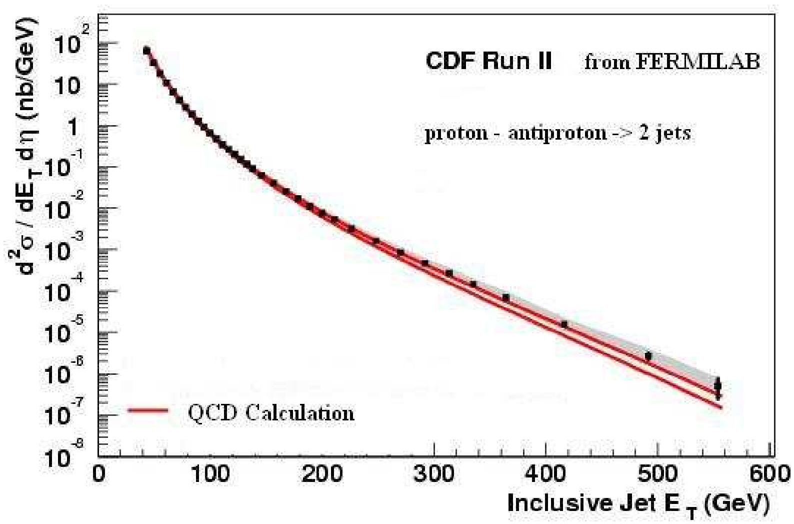
Now the total differential cross-section for proton-antiproton scattering is obtained by summing over all possible parton types that can be pulled out of the incoming protons (quarks, antiquarks, gluons).

Thus we finally obtain an expression for the proton-proton differential cross-section

$$\frac{d\sigma_{pp}(s)}{dp_T} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f_p^i(x_1) f_{\bar{p}}^j(x_2) \frac{d\hat{\sigma}(x_1 x_2 s)}{dp_T},$$

where the sum over i, j means sum over all possible partons.

QCD calculations based on this factorisation theorem agree well with experiment.



Only a single parton from each hadron takes part in the parton scattering process. The other partons in the incoming hadrons finally fragment into hadrons, which are moving almost in the same direction as the incoming protons (and are usually not observed because they get lost in the beam-pipe of the accelerator).