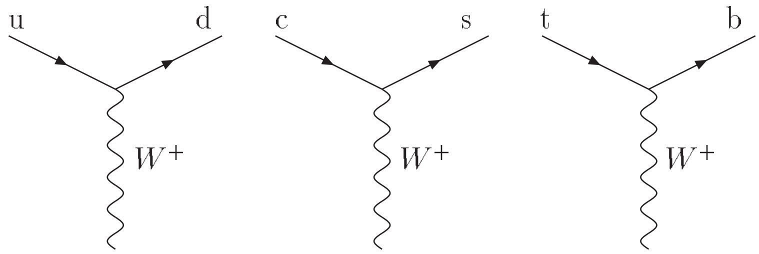
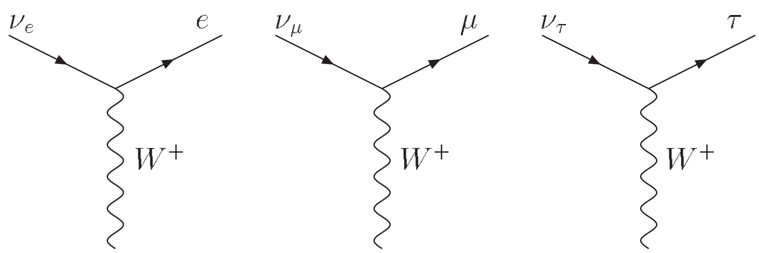
16 Weak Interactions

The weak interactions are mediated by $W^{\pm}$ or (neutral) Z exchange. In the case of $W^{\pm}$, this means that the flavours of the quarks interacting with the gauge boson can change.

$W^{\pm}$ couples to quark pairs (u, d). (c, s), (t, b) with vertices

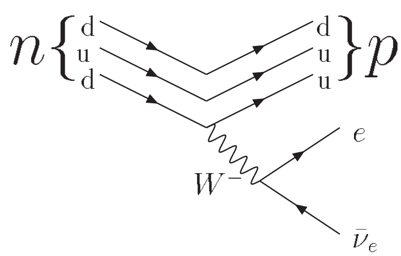


as well as to leptons (ν_e, e). (ν_μ, μ), (ν_τ, τ) with vertices



Note that in these interactions both quark number (baryon number) and lepton number are conserved.

It is this process that is responsible for β-decay. Neutron decays into a proton because a d-quark in the neutron converts into a u-quark emitting a W^- which then decays into an electron and anti-neutrino.



The amplitude for such a decay is proportional to

$$\frac{g_W^2}{(q^2 - M_W^2 c^2)},$$

where g_W is the strength of the coupling of the W^- to the quarks or leptons and $q^2 = E_q^2/c^2 - |\mathbf{q}|^2$, where $\mathbf{q}$ is the momentum transferred between the neutron and proton and E_q

is the energy transferred. This momentum is of order 1 MeV/c and so we can neglect it in comparison with $M_W c$ which is 80.4 GeV/c. Thus the amplitude is proportional to

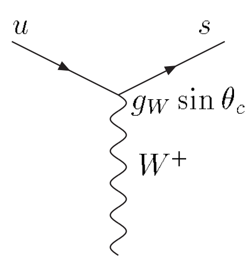
$$\frac{g_W^2}{M_W^2 c^2}.$$

The coupling g_W is not so small. In fact it is twice as large as the electron charge e. Weak interactions are weak because of the large mass term in the denominator.

At modern high energy accelerators, it is possible to produce weak interaction processes in which $|\mathbf{q}| \sim M_W c$ or even $|\mathbf{q}| \gg M_W c$. In such cases weak interactions are larger than electromagnetic interactions and almost comparable with strong interactions.

16.1 Cabibbo Theory

Particles containing strange quarks, e.g. $K^\pm$, K^0, Λ etc. cannot decay into non-strange hadrons via the strong interactions, which have to conserve flavour, but they can decay via the weak interactions. This is possible because $W^\pm$ not only couples a u-quark to a d-quark but can also (with a weaker coupling) couple a u-quark to an s-quark so we have a vertex



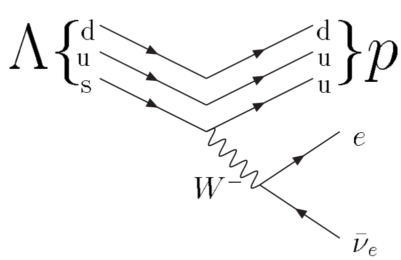
with coupling $g_W \sin\theta_C$, whereas the $u-d-W$ coupling is actually $g_w \cos\theta_C$. θ_C is called the "Cabibbo angle" and its numerical value is $\sin\theta_C \approx 0.22$.

This coupling allows a strange hadron to decay into non-strange hadrons and (sometimes) leptons.

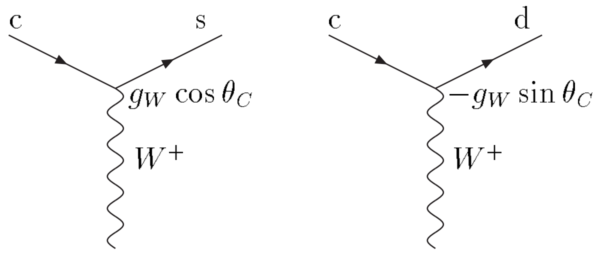
Thus, for example the decay

$$\Lambda \rightarrow p + e^- + \bar{\nu}_e$$

occurs when an s-quark converts into a u quark and emits a W^- which then decays into an electron and anti-neutrino. The Feynman graph is



Likewise, the c-quark has a coupling to the s-quark with coupling $g_W \cos\theta_C$ and a coupling to a d-quark with coupling $-g_W \sin\theta_C$.



This implies that charm hadrons are more likely to decay into hadrons with strangeness, because the coupling between a c-quark and a s-quark is larger than between a c-quark and a d-quark.

We can piece this together in a matrix form as follows

$$g_W \begin{pmatrix} d & s \end{pmatrix} \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}$$

This 2×2 matrix is called the "Cabbibo matrix". It is described in terms of a single parameter, the Cabibbo angle.

Since we know that there are, in fact, three generations of quarks this matrix is extended to a general 3×3 matrix as follows

$$g_W \begin{pmatrix} d & s & b \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

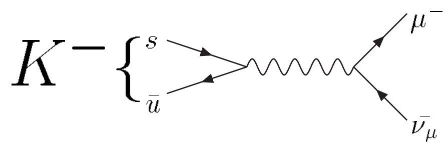
The 3×3 matrix is called the "CKM" (Cabibbo, Kobayashi, Maskawa) matrix. Quantum-mechanical constraints lead to the conclusion that of the nine elements there are only four independent parameters. Comparing the CKM matrix with the Cabibbo matrix we see that to a very good approximation, $V_{ud} \approx V_{cs} \approx \cos\theta_C$ and $V_{us} \approx -V_{cd} \approx \sin\theta_C$.

16.2 Leptonic, Semi-leptonic and Non-Leptonic Weak Decays

Because the $W^\pm$ couples either to quarks or to leptons, decays of strange mesons can either be leptonic, meaning that the final state consists only of leptons, semi-leptonic, meaning that the final state consists of both hadrons and leptons, or non-leptonic, meaning that the final state consists only of hadrons. For strange baryons only semi-leptonic and non-leptonic decays are possible because baryon number is strictly conserved - so there must be a baryon in the final state. Lepton number is also strictly conserved which means that a charged lepton is always accompanied by its anti-neutrino (or vice versa) in the final state.

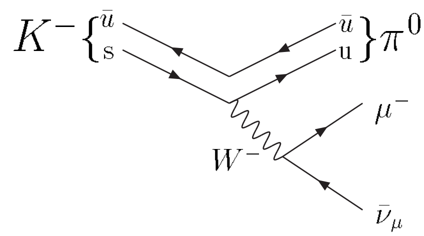
For mesons, examples are:

$$\text{Leptonic decay } K^- \rightarrow \mu^- + \bar{\nu}_\mu$$

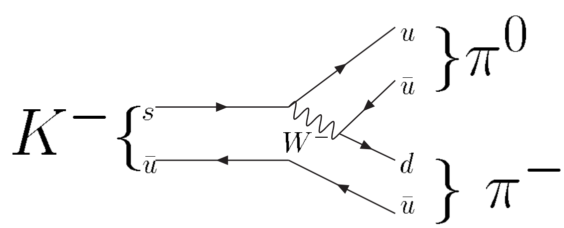


As well as converting an s-quark into a u-quark to emit a W^-, it is also possible to create a W^- from the annihilation of an s-quark with a $\bar{u}$ anti-quark.

Semi-leptonic decay $K^- \rightarrow \mu^- + \bar{\nu}_\mu + \pi^0$



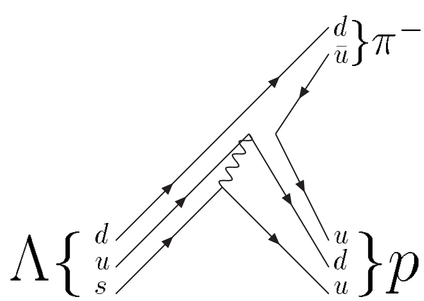
Non-leptonic decay $K^- \rightarrow \pi^0 + \pi^-$



Note that $m_K > 2m_\pi$ which is why this non-leptonic decay mode is energetically allowed.

In the case of baryons, we have already seen an example of a semi-leptonic decay, $\Lambda \rightarrow p\, e^-\ \bar{\nu}_e$. An example of a non-leptonic decay is

$$\Lambda \rightarrow p\, \pi^-$$



A W^- is exchanged between the s-quark and the u-quark in the Λ, converting them into a u-quark and a d-quark respectively. A $u - \bar{u}$ quark-antiquark pair is created in the process in order to make up the final state hadrons of a proton and a negative pion.

16.3 Flavour Selection Rules in Weak Interactions

Since in the exchange of a single $W^{\pm}$ an s-quark can be converted into a non-strange quark, it is highly unlikely that two strange quarks would be converted into non-strange quarks in the same decay process. We therefore have a selection rule for weak decay processes

$$\Delta S \; = \; \pm 1$$

Therefore, hadrons with strangeness -2 which decay weakly must first decay into a hadron with strangeness -1 (which in turn decays into non-strange hadrons). Thus, for example, we have

$$\Xi^0 \; \rightarrow \; \Lambda \, + \, \pi^0$$

The same selection rules apply for changes in other flavours (charm, bottom).

16.4 Parity Violation

The parity violation observed in β-decay arises because the $W^{\pm}$ tends to couple to quarks or leptons, which are left-handed (negative helicity), i.e. states in which the component of spin in their direction of motion is $-\frac{1}{2}\hbar$.

$W^{\pm}$ always couple to left-handed neutrinos. For quarks and massive leptons the $W^{\pm}$ *can* couple to positive helicity (right-handed) states, but the coupling is suppressed by a factor

$$\frac{mc^2}{E},$$

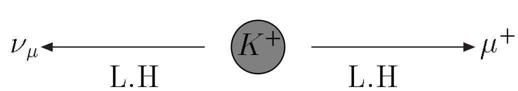
where m is the particle mass and E is its energy. The suppression is much larger for relativistically moving particles

In the case of nuclear β-decay, the nucleus is moving non-relativistically, but the electron typically has energy of a few MeV (and a mass of 0.511 MeV/c^2), so there is a significant suppression of the coupling to right-handed electrons. This is what was observed in the experiment by C.S. Wu on ^{60}Co.

For the coupling of $W^{\pm}$ to anti-quarks or anti-leptons, the helicity is reversed -i.e. the $W^{\pm}$ always couples to positive helicity anti-neutrinos and usually to positive helicity e^+, μ^+, τ^+ or to antiquarks, with a suppressed coupling to left-handed antileptons or anti-quarks.

A striking example of the consequence of this preferred helicity coupling can be seen in the leptonic decay of K^+.

$$K^+ \; \rightarrow \; \mu^+ \, + \, \nu_\mu$$



In the rest frame of the K^+ the momentum is zero, so the μ^+ and the ν_μ must move in opposite directions. The K^+ has zero spin, so by conservation of angular momentum, the two decay particles must have opposite spin component in any one chosen direction (e.g. the direction of the μ^+. This means that they have the *same* helicity. This means that the $W^\pm$ couples to the left-helicity anti-muon, μ^+ and such a coupling is suppressed by

$$\frac{m_\mu c^2}{E_\mu}$$

If we look at the decay mode

$$K^+ \rightarrow e^+ + \nu_e,$$

the same argument would lead to a suppression (of the decay amplitude) of

$$\frac{m_e c^2}{E_e}.$$

Since $m_e \ll m_\mu$ we expect the decay into a positron to be heavily suppresses. In fact we expect the ratio of the partial widths

$$\frac{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu)}{\Gamma(K^+ \rightarrow e^+ \nu_e)} = \frac{m_\mu^2}{m_e^2} \approx 4 \times 10^4$$

This coincides very closely to the experimentally observed ratio.

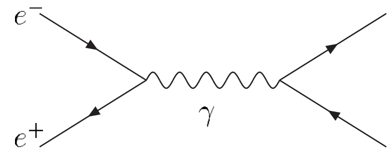
16.5 Z-boson interactions

As well as exchange of $W^\pm$ in which flavour is changed, the weak interactions are also mediated by a neutral gauge-boson, Z. This couples to both quarks and leptons but does not change flavour.

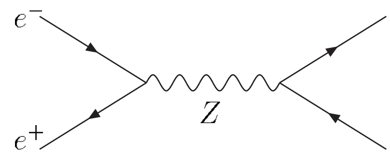
In that sense the interactions of the Z are similar to that of the photon, but there are some important differences.

- The Z couples to neutrinos whereas the photon does not (neutrinos have zero electric charge).
- The Z has a mass of 91.1 Gev/c^2, so the interactions are short range - like the interactions of the $W^\pm$.
- The Z also has a coupling of different strength to left-handed (negative helicity) and right-handed (positive helicity) quarks and leptons and so these interactions also violate parity.

Nevertheless, in any process where there can be photon exchange, there can also be Z exchange. In terms of Feynman diagrams for $e^+ e^-$ scattering into any pair of final state particles, we have



but also



The first diagram (photon exchange) has a propagator

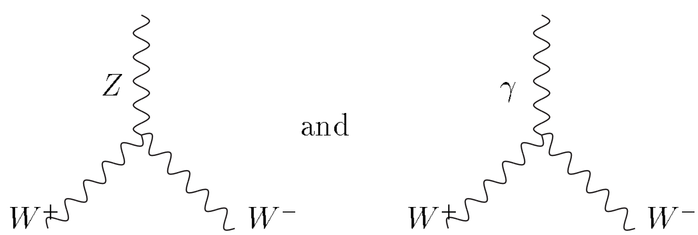
$$1/s,$$

where $\sqrt{s}$ is the centre-of-mass energy, whereas the second diagram (Z exchange) has a propagator

$$\frac{1}{s - M_Z^2 c^4}.$$

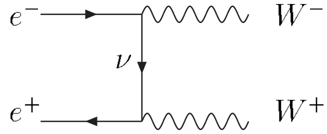
For relatively low centre-of-mass energies for which $\sqrt{s} \ll M_Z c^2$, the second diagram may be neglected and the second diagram gives a negligible contribution. But as $\sqrt{s}$ grows to become comparable (or greater than) $M_Z c^2$ both of these diagrams are equally important.

The Z and photon can both couple to $W^\pm$, so we get interaction vertices

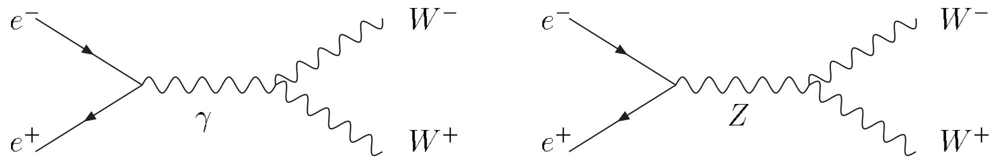


The interaction between the photon and $W^\pm$ is not surprising since the $W^\pm$ are charged and we would expect them to interact with photons, with coupling e. The interaction of $W^\pm$ with the Z is similar but has a different coupling.

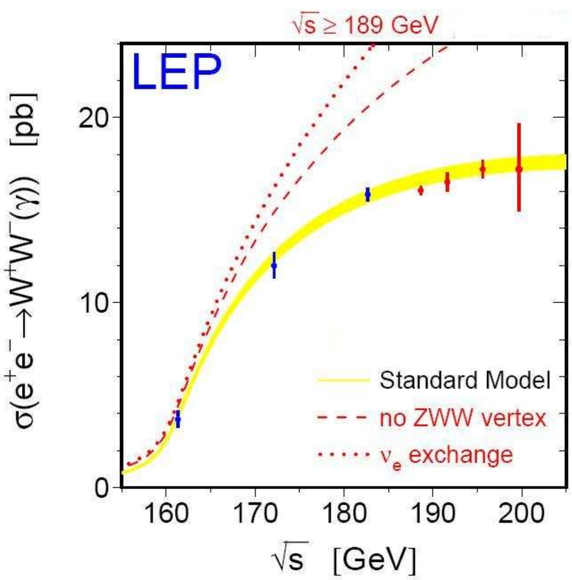
The coupling of the Z and photon to the $W^\pm$ was confirmed at the LEPII experiment at CERN where it was possible to accelerate electrons and positrons to sufficient energies to produce a W^+ and a W^- in the final state. From the coupling of the W to electron and neutrino the Feynman diagram for this process is



but because of the coupling of the Z and photon to $W^{\pm}$ we also have diagrams



The data from LEPII clearly show that these graphs have to be taken into account



It turns out that the Standard Model of weak and electromagnetic ("electroweak") interactions, developed in the 1960's by Glashow, Weinberg, and Salam, gives a relation between the weak coupling g_W, the (magnitude of the) electron charge, e and the masses of the Z and $W^{\pm}$

$$\frac{M_W}{M_Z} = \cos\theta_W$$

where θ_W is known as the weak mixing angle.

$$e = g_W \sin\theta_W = g_W \sqrt{1 - \frac{M_W^2}{M_Z^2}}$$

This enables us to make an order of magnitude estimate of the rates for weak processes at low energies.

At energies $\ll M_W c^2$, the amplitude for a $W^{\pm}$ exchange process is proportional to

$$\frac{g_W^2}{4\pi\epsilon_0 M_W^2 c^4},$$

so that the rate is proportional to

$$\left(\frac{g_W^2}{4\pi\epsilon_0 M_W^2 c^4}\right)^2.$$

Now for a weak decay rate we want dimensions of inverse time, so we need to multiply this by something with dimensions of the fourth power of energy divided by time. The only quantity proportional to the energy is the Q value of the decay, Q_β and to get inverse time we can divide by $\hbar$ so we get an estimate

$$\text{Rate} \sim \left(\frac{g_W^2}{4\pi\epsilon_0\hbar c M_W^2 c^4}\right)^2 . \frac{Q_\beta^5}{\hbar}$$

The pre-factor is actually quite small. For example, for muon decay $Q_\beta \approx m_\mu c^2$, and the muon decay rate is actually

$$\frac{1}{\tau_\mu} = \frac{1}{768\pi^3}\left(\frac{g_W^2}{4\pi\epsilon_0\hbar c}\right)^2 \frac{m_\mu^4}{M_W^4}\frac{m_\mu c^2}{\hbar}.$$

We know

$$\frac{g_W^2}{4\pi\epsilon_0\hbar c} = \frac{e^2}{4\pi\epsilon_0\hbar c \sin^2\theta_W} = \frac{\alpha}{\sin^2\theta_W}$$

and

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}.$$

Therefore from the measured masses of the W and Z we can determine the muon lifetime.

16.6 The Higgs mechanism

There is one further particle predicted by the Standard Model of electroweak interactions which has not yet been discovered.

This arises from the mechanism, discovered by P.Higgs, by which particles acquire their mass. The basic idea is that there exists a field, ϕ called the “Higgs field” which has a constant non-zero value everywhere in space. This constant value is called the “vacuum expectation value”, $\langle\phi\rangle$.

In the absence of this field it is assumed that all particles would be massless and would travel with velocity c. But because of their interaction with the background Higgs field they are slowed down - thereby acquiring a mass, M

$$M = \frac{1}{2}\frac{g_H}{\sqrt{\epsilon_0\hbar c}}\langle\phi\rangle,$$

where g_H is the coupling of the particle to the Higgs field (the denominator factor $\sqrt{\epsilon_0\hbar c}$ gives it the correct dimensions.) This mechanism is part of the Standard Model.

The Higgs field couples to $W^\pm$ with coupling g_W so that

$$M_W = \frac{1}{2}\frac{g_W}{\sqrt{\epsilon_0\hbar c}}\langle\phi\rangle.$$

Inserting $g_W = e/\sin\theta_W$ with $\cos\theta_W = M_W/M_Z$ and $M_W = 80.4\,\mathrm{GeV/c^2}$, and $M_Z = 91.2\,\mathrm{GeV/c^2}$, we get the value of the vacuum expectation value

$$\langle\phi\rangle = 250\,\mathrm{GeV/c^2}$$

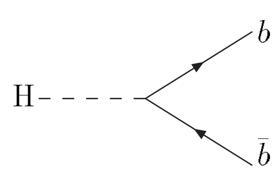
Other particles couple to the Higgs field with couplings that are proportional to their mass.

In the same way that there are quanta of the electromagnetic field which are particles (photons), so there must be quanta of the Higgs field. These are called "Higgs particles". They must necessarily exist if the Higgs mechanism for generating masses for particles is to be consistent with quantum physics.

Although these particles have so far not been discovered, we expect to see them soon after the LHC starts running. So far we know the following about these particles (assuming the Standard Model to be correct).

1. They have spin zero. This follows from the fact that the vacuum expectation value has to be invariant under Lorentz transformations - so that it is the same in all frames of reference.
2. They couple to $W^\pm$ and Z (which are consequently massive).
3. They do *not* couple to photons (which are massless) so they are uncharged.
4. They do not couple to gluons (which are massless) and so they do not take part in the strong interactions.
5. Their coupling to massive particles is proportional to the particle mass.
6. Their mass is greater than 115 $\mathrm{GeV/c^2}$ (they were not seen at LEPII).
7. It is expected that their mass is less than 200 $\mathrm{GeV/c^2}$.

Unless the mass of the Higgs particle is almost at the limit of item (7) above, they will not be sufficiently massive to decay into W^+W^- or two Z particles. They will decay into quark-antiquark or lepton-antilepton pairs. They will therefore decay into the most massive quark-antiquark flavour possible, because their coupling to quarks is proportional to the mass. The t-quark mass is 175 $\mathrm{GeV/c^2}$ so that they cannot decay into a $t-\bar{t}$ pair. The next most massive quark is the b-quark and so we expect the decay to be predominantly into a $b-\bar{b}$ pair.



The b-quark and $\bar{b}$ antiquark are not observed directly, but they will convert into two jets of hadrons, one containing at least one hadron with a b-quark and the other containing at least one hadron with a $\bar{b}$ antiquark. This will be a clean signal for Higgs decay.