Renormalisation Group Running and Neutrino Sum Rules "UK Neutrino Network Meeting"

Talk is based on the paper (arXiv:0808.2782 [hep-ph])

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Introduction

- The latest data from neutrino oscillation experiments is in good agreement with Tri-bimaximal Mixing pattern (TBM),

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} P_{Maj}$$

- Since future experiments will be sensitive to small deviations from TBM, it is important to study the theoretical uncertainty in such predictions.

The PMNS matrix

$$
\begin{aligned}
U_{PMNS} &= V_{e_L} V_{\nu_L}^\dagger \\
&= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{23}c_{13} \end{bmatrix} P_{Maj}
\end{aligned}
$$

Another useful parametrisation of the PMNS matrix is:

$$
U_{PMNS} \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1 - \frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix} P_{Maj}.
$$

$$
s_{13} = \frac{r}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}}(1+s), \quad s_{23} = \frac{1}{\sqrt{2}}(1+a).
$$

$$
0 < r < 0.22, \quad -0.11 < s < 0.04, \quad -0.12 < a < 0.13.
$$

Neutrino mass

- Neutrino mass is zero in the SM for many reasons.
  - There are no right handed Neutrinos.
  - There are only Higgs doublets.
- To generate neutrino mass, one or more of these conditions must be relaxed.
- This gives three types of neutrino mass:
  - Majorana masses of the form $m_{LL}\bar{\nu}_L\nu_L^c$
  - Majorana masses of the form $M_{RR}\bar{\nu}_R\nu_R^c$
  - Dirac masses of the form $m_{LR}\bar{\nu}_L\nu_R$

The See-Saw mechanism

- The See Saw mechanism is one appealing explanation for the smallness of neutrino mass.
- The left handed Majorana masses (m_{LL}) are strictly forbidden in the standard model.
- For the simplest version of the see-saw mechanism, one can assume m_{LL} to be zero at first, but are effectively generated after introducing ν_R
- M_{RR} can be arbitrarily large.With these types of neutrino mass, the see-saw mass matrix is given as,

$$\begin{pmatrix} \bar{\nu}_L \bar{\nu}_R^c \end{pmatrix} \begin{pmatrix} 0 & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

Sequential dominance

- A simple way to achieve a neutrino mass hierarchy with large atmospheric and solar mixing is the idea of sequential dominance.
- Tri-bimaximal neutrino mixing can emerge from SD,

$$M_{RR} \approx \begin{pmatrix} Y & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & X' \end{pmatrix}$$

  We write the neutrino Yukawa matrix as,

$$Y^{\nu}_{LR} = \begin{pmatrix} d & a & a' \\ e & b & b' \\ f & c & c' \end{pmatrix}$$

- SD corresponds to the right-handed neutrino of mass Y being the dominant term while the right-handed neutrino of mass X giving the leading sub-dominant contribution to the see-saw mechanism.

$$\frac{|e^2|, |f^2|, |ef|}{Y} \gg \frac{|xy|}{X} \gg \frac{x' y'}{X'}$$

Constrained sequential dominance

- Tri-bimaximal mixing, in which $\tan\theta^\nu_{23} = 1$, $\tan\theta^\nu_{12} = 1/\sqrt{2}$ and $\theta^\nu_{13} = 0$ corresponds to the choice,

$$
\begin{aligned}
|a| &= |b| = |c|, \\
|d| &= 0, \\
|e| &= |f| \\
e^* b + f^* c &= 0.
\end{aligned}
$$

- We refer to these conditions as Constrained Sequential Dominance (CSD).
- For a supersymmetric theory with SO(3) family symmetry, the neutrino Yukawa matrix becomes,

$$
Y^\nu_{LR} = \begin{pmatrix} 0 & ae^{i\delta_2} & c_1 \\ ee^{i\delta_1} & be^{i\delta_2} & c_2 \\ fe^{i\delta_1} & ce^{i\delta_2} & c_3 \end{pmatrix},
$$

- To satisfy CSD, it is sufficient to have,

$$
\begin{aligned}
e &= -f \\
a &= b = c.
\end{aligned}
$$

Cabibbo-like Corrections

- We assume that the charged lepton mixing matrix has a Cabibbo-like structure and that it is dominated by a 1-2 mixing θ_{12}^E,

$$V_{e_L} \approx \left(\begin{array}{ccc} c_{\theta_{12}^E} & -s_{\theta_{12}^E} e^{-i\lambda_{12}^E} & 0 \\ s_{\theta_{12}^E} e^{i\lambda_{12}^E} & c_{\theta_{12}^E} & 0 \\ 0 & 0 & 1 \end{array} \right),$$

- The neutrino Yukawa matrix with TBM becomes transformed as,

$$Y_\nu \rightarrow Y'_\nu = V_{e_L} Y_\nu$$

$$\begin{aligned} U_{PMNS} &= V_{e_L} V_{\nu_L}^\dagger \\ &= \left[\begin{array}{ccc} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}e^{i\delta} & c_{23}c_{12} - s_{13}s_{23}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta} & -s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta} & c_{23}c_{13} \end{array} \right] P_{Maj} \end{aligned}$$

Cabibbo-like corrections and sum rules

- The elements $(U_{PMNS})_{31}$, $(U_{PMNS})_{32}$ and $(U_{PMNS})_{33}$ are unaffected by the Cabibbo-like charged lepton correction.

$$
\begin{aligned}
|(U_{PMNS})_{31}| &\approx |(V^{\dagger}_{\nu_L})_{31}| \approx \frac{1}{\sqrt{6}} \\
|(U_{PMNS})_{32}| &\approx |(V^{\dagger}_{\nu_L})_{32}| \approx \frac{1}{\sqrt{3}} \\
|(U_{PMNS})_{33}| &\approx |(V^{\dagger}_{\nu_L})_{33}| \approx \frac{1}{\sqrt{2}}
\end{aligned}
$$

- We define third row deviation parameters ξ_i:

$$
\begin{aligned}
\xi_1 &\equiv \sqrt{6}\,|(U_{PMNS})_{31}| - 1 \equiv \sqrt{6}\,|s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta}| - 1, \\
\xi_2 &\equiv \sqrt{3}\,|(U_{PMNS})_{32}| - 1 \equiv \sqrt{3}\,|-s_{23}c_{12} - s_{13}c_{23}s_{12}e^{i\delta}| - 1, \\
\xi_3 &\equiv \sqrt{2}\,|(U_{PMNS})_{33}| - 1 \equiv \sqrt{2}\,|c_{23}c_{13}| - 1.
\end{aligned}
$$

- For Cabbibo like corrections, these deviation parameters are all zero ($\xi_i = 0$)

Cabibbo-like corrections and sum rules

- Assuming neutrino tri-bimaximal mixing. We get

$$\Gamma_1 \equiv \arcsin\left(\sqrt{2}\,|s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta}|\right) \approx 35.26\,^\circ,$$

- The sum rule can be simplified further to leading order in s_{13},

$$\Gamma_2 \equiv \arcsin\left(\sqrt{2}\,(s_{23}s_{12} - s_{13}c_{23}c_{12}\cos\delta)\right) \approx 35.26\,^\circ.$$

- Combining with $|(V^\dagger_{\nu_L})_{33}| \approx |(U_{PMNS})_{33}| \approx \frac{1}{\sqrt{2}}$, we get

$$\Gamma_3 \equiv \theta_{12} - \theta_{13}\cos(\delta) \approx 35.26\,^\circ.$$

- The above sum rule can be expressed in terms of TB deviation parameters as,

$$\sigma_1 = r\cos\delta - s = 0.$$

- To deal with issues of canonical normalisation corrections, the following sum rule has been proposed:

$$\sigma_2 = r\cos\delta + \frac{2}{3}a - s = 0.$$

(Antusch, King and Malinsky)

Numerical Example

- Assume MSSM with $\tan(\beta) = 50$. The Majorana mass M_{RR} ($M_3 = 10^{16}$ GeV):

$$M_{RR} = \begin{pmatrix} 5 \times 10^{-9} & 0 & 0 \\ 0 & 5.78 \times 10^{-9} & 0 \\ 0 & 0 & 1 \end{pmatrix} M_3.$$

- Taking Y^{ν}_{LR} of the form

$$Y^{\nu}_{LR} = \begin{pmatrix} 0 & 1.061667\, be^{i\delta_2} & c_1 \\ ee^{i\delta_1} & be^{i\delta_2} & c_2 \\ -0.9799\, ee^{i\delta_1} & be^{i\delta_2} & c_3 \end{pmatrix},$$

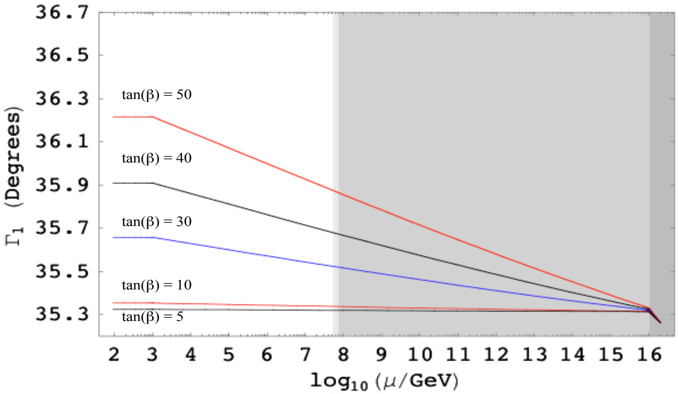
  $b = 8.125 \times 10^{-5}$, $e = 2.125 \times 10^{-4}$, $c_1 = 10^{-3}$, $c_2 = 0$ and $c_3 = 0.5809$. This gives exact TB mixing with ($\theta^{\nu}_{12} = 35.26\,^{\circ}$, $\theta^{\nu}_{23} = 45.00\,^{\circ}$, $\theta^{\nu}_{13} = 0.00\,^{\circ}$).

- If the CSD conditions were imposed exactely, we get $\theta_{12} = 33.97\,^{\circ}$, $\theta_{23} = 44.38\,^{\circ}$, $\theta_{13} = 0.059\,^{\circ}$

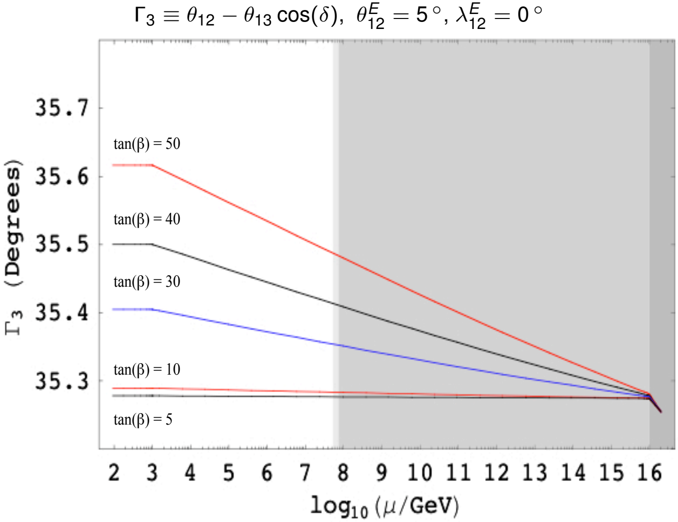
RG corrections to Γ_1

- To study the running of the neutrino mixing parameters from the GUT scale to the electroweak scale, the Mathematica package REAP was used (Antusch et al).

$$\Gamma_1 \equiv \arcsin\left(\sqrt{2}\,|s_{23}s_{12} - s_{13}c_{23}c_{12}e^{i\delta}|\right),\ \theta^E_{12} = 5^\circ,\ \lambda^E_{12} = 0^\circ$$



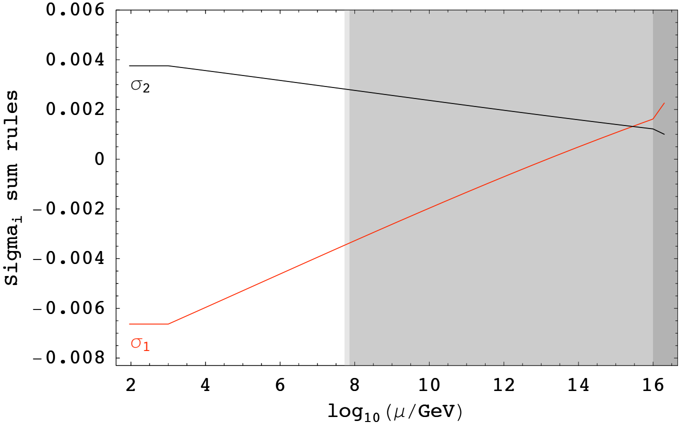
RG corrections to Γ_3



RG corrections to σ_i

$$\sigma_1 = r\cos\delta - s = 0, \quad \sigma_2 = r\cos\delta + \frac{2}{3}a - s = 0.$$

$$\theta_{12}^E = 5^\circ, \lambda_{12}^E = 30^\circ$$



More general charged lepton corrections

- We consider a more general charged lepton matrix given by,

$$V_{e_L} \approx \begin{pmatrix} c_{\theta^E_{12}} & -s_{\theta^E_{12}} e^{-i\lambda^E_{12}} & 0 \\ s_{\theta^E_{12}} e^{i\lambda^E_{12}} & c_{\theta^E_{12}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & c_{\theta^E_{23}} & -s_{\theta^E_{23}} e^{-i\lambda^E_{23}} \\ 0 & s_{\theta^E_{23}} e^{i\lambda^E_{23}} & c_{\theta^E_{23}} \end{pmatrix},$$

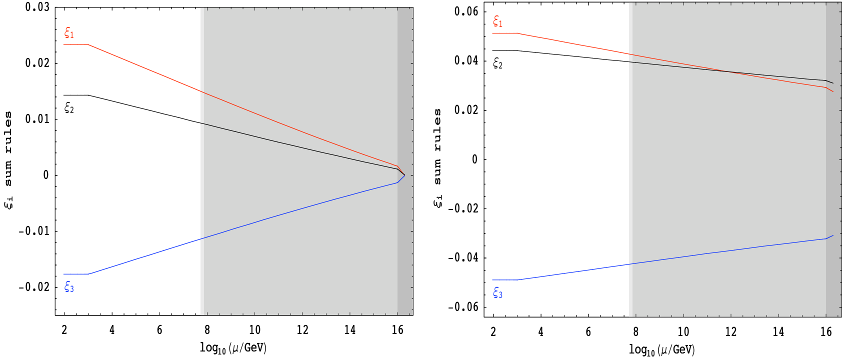
- The neutrino Yukawa matrix can be transformed as,

$$Y_\nu \to Y'_\nu = V_{e_L} Y_\nu .$$

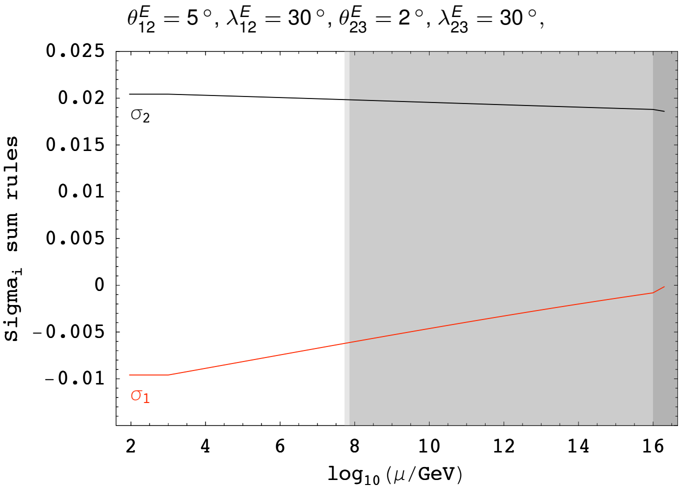
- For this case the ξ_i parameters are not exactly zero at the GUT scale.

RG corrections to ξ_i

$$\theta_{12}^E = 5^\circ,\ \lambda_{12}^E = 30^\circ,\ \theta_{23}^E = (0,2)^\circ,\ \lambda_{23}^E = (0,30)^\circ,$$



RG corrections to σ_i



Conclusions

- We have introduced a variety of sum rules in terms of neutrino mixing angles and TB deviation parameters.
- Cabibbo-like corrections are added to TBM at the GUT scale but since the parameters are measured experimentally at low energy scale, we need to look at RG running.
- The sum rules were found to be good for Cabbibo-like corrections (Γ_3 very good about 0.4°) with small deviations at the electroweak scale.
- For non Cabbibo-like charged lepton corrections, the sum rule σ_1 is found to be insensitive to θ_{23}^E.

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