The E_6SSM with a discrete non-Abelian family symmetry

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UK Neutrino Meeting 09

JHEP 0805:008,2008.
arXiv:0802.1909 [hep-ph]

Outline

- Purpose of Talk
- Introduction to the E_6SSM
- Explanation for the origin of neutrino masses and lepton mixing angles using the E_6SSM and a discrete non-Abelian family symmetry.
  - Solution to the 'flavour problem'
- Summary

Introduction

- The E_6SSM is a low-energy supersymmetric standard model which originates from a broken high-energy E_6 symmetry. (S.F.King, R. Nevzorov , S. Moretti)
- It extends the MSSM to solve the μ-problem and the little fine-tuning problem of the MSSM.
- Because of this the E_6SSM contains the necessary ingredients for small neutrino masses
- But it lacks a natural explanation for the observed large lepton mixing angles.

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- Because of this the E_6SSM contains the necessary ingredients for small neutrino masses

- But it lacks a natural explanation for the observed large lepton mixing angles.

- We extend the E_6SSM using a simple horizontal discrete symmetry to explain the origin of the large lepton mixing angles.

- The horizontal symmetry also solves the 'flavour problem': the origin for individual quark and lepton masses and CKM matrix elements.

Introduction to the E_6SSM

- Principal Motivation of the E_6SSM : to solve the μ-problem of the MSSM

- Why free parameter μ in $W_{MSSM} = \mu h_u h_d$ takes value $M_{SUSY} \sim 1\ TeV$

  (which is required for natural EWSB and for Higgsino mass)

- Since μ is a dimensional parameter we might expect it to be of order a physical cut-off scale such as M_{GUT} or M_p

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- Furthermore μ is SUSY-respecting, so unrelated to M_{SUSY} in the MSSM.
- Popular solution: forbid $\mu h_u h_d$ and introduce a SM singlet field S
  that couples to Higgs fields by: $\lambda_S S h_u h_d$
- $\mu = \lambda_S < S >$ so no longer a dimensional parameter in W
  (and $< S >$ related to soft mass of S)

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- But extended MSSM superpotential now contains a global $U(1)$ symmetry (PQ symmetry)

- One way to avoid an unwanted axion from $< S >$ is to couple S to a new
  local $U(1)$ symmetry: generates an observable Z'

Introduction to the E_6SSM

- The local $U(1)$ symmetry automatically forbids $\mu h_u h_d$ and its D-terms stabilize the potential:

$$V = m_S^2|s|^2 + \frac{g'^2}{2}(Q_S^2|s|^2)^2$$

$s \equiv \; < S > /\sqrt{2}$

$Q_S = U(1)$ charge of S

$m_S =$ Soft mass of S

- $\Rightarrow s^2 = -2m_S^2/g'^2 Q_s^2$ with $\mu = \lambda_S < S >$

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- However the local $U(1)$ group must be anomaly free

- E_6SSM : $U(1)$, called $U(1)_N$ (and SM gauge group) comes from an E_6 symmetry.

$$E_6 \rightarrow SO(10) \times U(1)_\psi$$
$$\rightarrow SU(5) \times \times U(1)_\chi \times U(1)_\psi$$
$$\rightarrow SM \times \times U(1)_N$$

- For $U(1)_N$ to be anomaly free, we must have complete fundamental 27 multiplets of E_6 at low-energies.

The E_6SSM

- One 27 contains one family of quarks and leptons (and Higgs Fields)
- $E_6 \rightarrow SO(10) \quad 27 \rightarrow 16 + 10 + 1$

$$27 = Q + u^c + d^c + L + e^c + \nu^c + h_u + h_d + D + \overline{D} + S$$

$\Rightarrow$ require 3 copies:

$$3\times 27 = MSSM + 3\times(D+\overline{D}) + 2\times(h_u + h_d) + 3\times S + 3\times \nu^c$$

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$\Rightarrow$ All 3×27 states must therefore survive below $< S > \sim TeV$ except for ν^c

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- Also require additional EW doublet states $H', \overline{H}'$ for gauge coupling unification at GUT scale:

$3 \times 27 + H', \overline{H}' = MSSM + 3 \times (5 + \overline{5} + 1)$ of $SU(5)$

(Adding complete $SU(5)$ multiplets to MSSM preserves GU)

- For the minimal version of the E_6SSM , the ME_6SSM

$H', \overline{H}'$ is substituted for an intermediate Pati-Salam symmetry (RJH, S.F. King)

Gauge coupling unification then moved closer to Planck scale.

The E_6SSM

- Sh_uh_d and Yukawa terms automatically from $27 \times 27 \times 27$ decomposition of E_6

$$W_{E_6SSM} = \lambda^S_{ijk} S^i h^j_u h^k_d + \lambda^u_{ijk} Q^i u^{cj} h^k_u + \lambda^d_{ijk} Q^i d^{cj} h^k_d + \lambda^e_{ijk} L^i e^{cj} h^k_d + \lambda^\nu_{ijk} L^i \nu^{cj} h^k_u$$

- Only third generation of Higgs $h^3_u,\ h^3_d$ taken to get EW VEV

  Breaking EW symmetry and giving mass to quarks and leptons.

- Third generation of S is taken as the field that gives the effective μ-term: $\lambda^S_{333} h^3_u h^3_d$

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- $27 \times 27 \times 27$ also contains terms involving non-MSSM states $D, \overline{D}$

  e.g. $h^Q_{ijk} D^i Q^i Q^j + h^D_{ijk} \overline{D}^i Q^j L^k$

- To prevent these from mediating rapid proton decay, a Z_2 symmetry is added to some states

  e.g. Z_2^L : only leptons are odd Z_2^B :leptons and $D, \overline{D}$ are odd

- The proton decay symmetries do not respect $E_6 \Rightarrow E_6$ symmetry broken.

The Neutrino Sector of the E_6SSM

- E_6SSM pre-built for small neutrino mass:
  - 27 of E_6 contains a right-handed neutrino
  - $U(1)_N$ defined so that ν_R has no charge.
  - $27 \times 27 \times 27$ contains $\lambda^{\nu}_{ijk} L_i \nu^c_j h_{uk}$

The Neutrino Sector of the E_6SSM

- E_6SSM pre-built for small neutrino mass:
  - 27 of E_6 contains a right-handed neutrino
  - $U(1)_N$ defined so that ν_R has no charge.
  - $27 \times 27 \times 27$ contains $\lambda^{\nu}_{ijk} L_i \nu^c_j h_{uk}$
- $U(1)_N$ allows for a Majorana mass term for ν_R : $M^{ij}_{RR} \overline{\nu}^i_R \nu^{cj}_R$

$\Rightarrow$ Can use a conventional see-saw mechanism with ν_R at a high-energy scale to explain small neutrino masses

e.g. Type I see-saw: $$M_{LL} = < h_u >^2 \lambda^{\nu} M^{-1}_{RR} (\lambda^{\nu})^T$$

- Physical neutrino masses = eigenvalues of M_{LL}
- e.g. $M_{RR} \sim \mathcal{O}(GUT)$

  $< h_u > \lambda^{\nu} \sim \mathcal{O}(M_W)$

  $\Rightarrow M_{LL} \sim \mathcal{O}(10^{-3} eV)$ which looks good for solar neutrinos.

Tri-bi-maximal Mixing

- Can use the see-saw mechanism in E_6SSM to explain small neutrino masses.
- But no natural explanation for origin of large lepton mixing angles.
- Lepton mixing matrix: $V_{PMNS} = V_{eL} V^{\dagger}_{\nu(L)}$

$$< h_d > V_{eL} Y^e V^{\dagger}_{eR} = diag(m_e, m_\mu, m_\tau)$$

$$V_{\nu L} m^{\nu}_{LL} V^{T}_{\nu L} = diag(m_1, m_2, m_3)$$

- Data consistent with a tri-bi-maximal form: $V_{PMNS} \sim \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

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- This can be explained using see-saw by constrained sequential dominance: (S.F. King)
  - Assumes one ν_R (e.g. ν_R^1) dominates see-saw mechanism and determines atmospheric neutrino mass and mixing angle.
  - another ν_R (e.g. ν_R^2) contributes sub-dominantly and determines the solar neutrino mass and mixing angle.

- In basis where M_{RR} is diagonal: $M_{RR} = diag(M_1, M_2, M_3)$

  λ^ν must then have following form to explain tri-bi-maximal mixing:

$$\lambda^\nu = \begin{pmatrix} 0 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{pmatrix}$$

with:
$$|A_2| = |A_3|$$
$$|B_1| = |B_2| = |B_3|$$
$$A_2 B_2 + A_3 B_3 = 0$$

- Take ν_R^1 to dominate the see-saw mechanism. e.g. take $M_1 \ll M_2 \ll M_3$

$$M_{LL} = < h_u >^2 \lambda^\nu M_{RR}^{-1} (\lambda^\nu)^T$$

- ν_R^1 Effectively couples to first column vector $(\nu_\mu - \nu_\tau)$, giving maximal atmospheric angle
- ν_R^2 Effectively couples to second column vector $(\nu_e + \nu_\mu + \nu_\tau)$, giving solar neutrino angle

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- $$M_{LL} \sim \frac{A^2}{M_1} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + \frac{B^2}{M_2} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\theta_{23} = 45^o$$
$$\theta_{13} = 0^o$$
$$\theta_{12} = 30^o$$

- $V_{\nu L} m_{LL}^\nu V_{\nu L}^T = diag(m_1, m_2, m_3)$

- $V_{PMNS} =$ Tri-bi-maximal in basis where charged lepton Yukawa matrix is diagonal

CSD from a Δ_{27} family symmetry

- One way to generate CSD is with a Δ_{27} family symmetry (G. Ross, S.F. King, I. Varzielas)

  where Δ_{27} is a non-Abelian discrete subgroup of $SU(3)$

  - Take Leptons to transform as 3 of $SU(3)$
  - and Higgs as trivial singlets.

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- Renormalizable $\lambda^{\nu}_{ijk} L_i \nu^c_j h_{uk}$ Yukawa term now forbidden by the horizontal symmetry

  Instead it is generated effectively from a higher-order operators:

$$\frac{\mathcal{O}(1)}{M_u^2} L_i \nu^c_j h_{uk} \overline{\phi}^i_{123} \overline{\phi}^j_{23} \rightarrow \lambda^{\nu}_{ij} L_i \nu^c_j h_{uk}$$

So that $\lambda^{\nu}_{ij} \sim < \overline{\phi} >^i_{123} < \overline{\phi} >^j_{23} / M_u^2$

- Where M_u is the mass scale of messenger particles.

  and $\overline{\phi}^i$ are scalar fields that transform as $\overline{3}$ and get VEVs in the following $SU(3)$ directions:

$$< \overline{\phi}^T_{123} > = \left(\begin{array}{ccc} b & b & b \end{array} \right) < \overline{\phi}^T_{23} > \propto \left(\begin{array}{ccc} 0 & a & -a \end{array} \right)$$

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- Above term then generates the following effective Yukawa matrix: $\lambda^{\nu} \sim \begin{pmatrix} 0 & ab & -ab \\ ab & 2ab & 0 \\ -ab & 0 & ab \end{pmatrix}$

- This is CSD in disguise:

CSD from a Δ_{27} family symmetry

$\longrightarrow$ The see-saw mechanism is invariant to non-unitary ν_R transformations: $\nu_R \rightarrow T\nu_R$

$$\lambda^\nu \rightarrow \lambda^\nu T^{-1} \qquad\qquad M_{RR} \rightarrow T^{T-1} M_{RR} T^{-1}$$

- With $T^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ λ^ν takes the form: $\lambda^\nu = \begin{pmatrix} 0 & B & C_1 \\ A & B & C_2 \\ -A & B & C_3 \end{pmatrix}$

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- This is the λ^ν matrix from Δ_{27} with $A = B = ab$

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- M_{RR} now has following off-diagonal form in this basis: $M_{RR} = \begin{pmatrix} M_1 & M_1 & 0 \\ M_1 & M_1 + M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix}$

- This can be generated by the Δ_{27} family symmetry using certain flavon fields.

  e.g. M_3 is generated by: $\frac{\mathcal{O}(1)}{M} \nu_i^c \nu_j^c H_R^i H_R^j$

  where H_R^i is from a $\overline{27}$ of E_6 and $\overline{3}$ of $SU(3)$ $\quad < H_R^T > \propto \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$

E_6SSM with Δ_{27} family symmetry

- The Δ_{27} symmetry also explains the origin of quark and charged lepton masses and and CKM matrix elements.

$\longrightarrow$ Quarks as well as leptons taken to transform as triplets.

- All Higgs-matter couplings in $27 \times 27 \times 27$ now forbidden by the horizontal symmetry

  They are instead generated effectively from higher-order operators:

  e.g. $\frac{\mathcal{O}(1)}{M_u^2} Q_i u_j^c h_{uk} \overline{\phi}^i \overline{\phi}^j \longrightarrow \lambda_{ij}^u Q_i u_j^c h_{uk}$

$$\Rightarrow \lambda_{ij}^u \sim <\overline{\phi}>^i <\overline{\phi}>^j /M_u^2$$

- $\overline{\phi}^i$ chosen to create a particular form of quark and lepton Yukawa matrices.

  These explain masses and mixing angles once h_u^3 and h_d^3 get VEVs.

Full Model

- Can apply Δ_{27} symmetry to the E_6SSM since E_6 is a broken symmetry

  E_6SSM naturally contains a neutrino see-saw mechanism

  $\Rightarrow$ (approximate) Tri-bi-maximal mixing can result from CSD.

- Δ_{27} explains the origin of quark and lepton masses and mixing angles for the E_6SSM

- Want to preserve the E_6SSM solution to the μ-problem:

  S^3 taken to be a singlet of Δ_{27} (like $h^3_{u,d}$)

  Therefore $\lambda^{333}_S S^3 h^3_u h^3_d$ term still allowed at the renormalizable level.

Full Model

- Full model:

Field	Δ_{27}	$SU(4)_{PS} \times SU(2)_L \times SU(2)_R$	$U(1)_R$	$U(1)$	Z_2	Z_2^H
F^i	3	$(4,2,1)$	1	0	+	+
F^{ci}	3	$(\overline{4},1,\overline{2})$	1	0	+	+
h^3	1	$(1,2,2)$	0	0	+	+
$h^{1,2}$	1	$(1,2,2)$	0	0	+	-
$\mathcal{D}^{1,2,3}$	1	$(6,1,1)$	0	0	+	+
$S^{1,2,3}$	1	$(1,1,1)$	2	0	+	+
H_R	$\overline{3}$	$(4,1,2)$	0	0	+	+
H_{45}	1	$(15,1,3)$	0	2	+	+
ϕ_{123}	3	$(1,1,1)$	0	-1	+	+
ϕ_3	3	$(1,1,1)$	0	3	+	+
ϕ_1	3	$(1,1,1)$	0	-4	-	+
$\overline{\phi}_3$	$\overline{3}$	$(1,1,2\times 2)$	0	0	-	+
$\overline{\phi}_{23}$	$\overline{3}$	$(1,1,1)$	0	-1	-	+
$\overline{\phi}_{123}$	$\overline{3}$	$(1,1,1)$	0	1	-	+

- Where $U(1)_R \times U(1) \times Z_2$ symmetries constrain the model.

- As well as explaining Higgs, quark, leptron masses,

  full theory also gives mass to rest of 3×27

  e.g. $D, \overline{D}$ get their mass from the renormalizable term: $\lambda_S^{Dij} S^3 D_i \overline{D}_j$

  $h^1_{u,d}$ get their mass from the renormalizable term: $\lambda_S^{311} S^3 h^1_u h^1_d$

Summary

- In the way that the E_6SSM solves the μ-problem of the MSSM, it naturally allows for a conventional neutrino see-saw mechanism.

- Applied a Δ_{27} family symmetry to E_6SSM so that together with a see-saw mechanism, this predicts tri-bi-maximal mixing using CSD.

- Δ_{27} also explains the origin of quark and charged lepton masses and CKM elements.

- Way in which Δ_{27} is applied preserves the E_6SSM solution to the μ-problem.

- Full theory = a low-energy supersymmetric model originating from a broken E_6 (vertical) symmetry and a Δ_{27} horizontal symmetry.

- Combines the virtues of both the E_6SSM and a Δ_{27} family symmetry.

- Provides an origin for the mass of the Higgs, quarks and leptons as well as their mixing angles.