Accidental A_4 family symmetry

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Orbifold GUT theories

- Extra dimensional theories
- Parities under Orbifolding are used to make particles heavy
- Multiple parities can be used

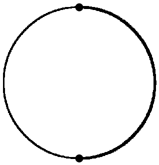
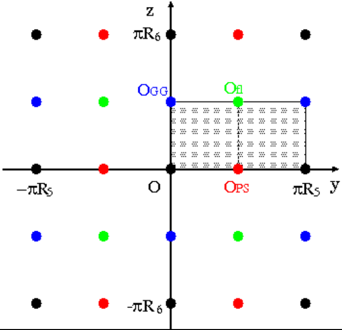


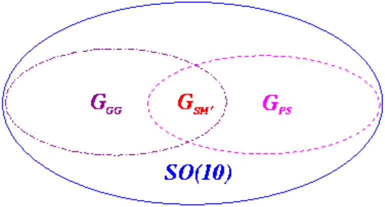
Figure: The fundamental domain is shown in bold. It lies between the two fixed points at 0 and πR

An Orbifold GUT theory

An SO(10) model

Asaka, Buchmüller, Covi. Gauge unification in six-dimensions. hep-ph/0108021





A_4 family symmetries

- A_4 can reproduce the TBM mixing
- $$U = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$
- Often additional symmetries are used e.g. U(1) and $\mathbb{Z}_N$

The A_4 group

- Subgroup of SO(3)
- Group of even permutations of 4 objects
- Can be generated by two permutations S and T
- $S : (1234) = (4321)$ and $T : (1234) = (2314)$
- $S^2 = T^3 = (ST)^3 = 1$
- Contains 3 singlets and a triplet, $1, 1', 1'', 3$

- Defining $z = x_5 + ix_6$
- The torus is generated by

$$z \longrightarrow z + 1$$
$$z \longrightarrow z + \gamma \qquad \gamma = e^{i\frac{\pi}{3}}$$

- The orbifolding is defined by

$$z \longrightarrow -z$$

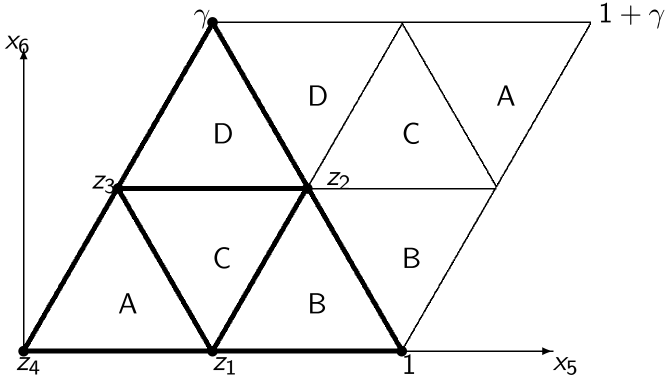
- We are left with a tetrahedron in the 2 extra dimensions
- The Tetrahedron can be generated by

$$\mathcal{S} : z \longrightarrow z + \frac{1}{2}$$
$$\mathcal{T} : z \longrightarrow \omega z \qquad \omega \equiv \gamma^2$$

- These two transformations permute the fixed points

$$\mathcal{S} : (z_1, z_2, z_3, z_4) \longrightarrow (z_4, z_3, z_2, z_1)$$
$$\mathcal{T} : (z_1, z_2, z_3, z_4) \longrightarrow (z_2, z_3, z_1, z_4).$$

The Orbifold



- Altarelli, Feruglio and Lin. Tri-bimaximal Neutrino Mixing from Orbifolding:hep-ph/0610165

The Model

- Based on SU(5) $\mathcal{N} = 1$ in 6d
- U(1) family symmetry
- A_4 comes from the symmetry of the extra dimensions left over from compactification

- Matter fields are brane fields confined to fixed points
- Assign the 10-plets $T_{1,2,3}$ to $1'', 1', 1$ of A_4
- RH neutrinos and 5-plets are 3-plets of A_4
- θ, θ'' break the U(1) symmetry
- The A_4 is broken by triplets φ_T and φ_S and a singlet ξ

Compactification

- We can expand our fields on the compact dimensions using
- $$\Phi(x, x_5, x_6) = \frac{1}{2\pi\sqrt{R_1 R_2 \sin\theta}} \sum_{m,n} \Phi^{(m,n)}(x) \exp\left\{ i \left(\frac{m}{R_1}\{x_5 - \frac{x_6}{\tan\theta}\} + \frac{n x_6}{R_2 \sin\theta} \right) \right\}$$
- We get a Kaluza-Klein states of mass $M(m,n) = \frac{1}{\sin\theta}\sqrt{\left(\frac{m}{R_1}\right)^2 + \left(\frac{n}{R_2}\right)^2 - \frac{2mn\cos\theta}{R_1 R_2}}$
- We are left with a $\mathcal{N} = 2$ SU(5) theory in 4d from the $\mathcal{N} = 1$ 6d theory

Getting rid of the unwanted SUSY

- We have a vector multiplet (V_μ, λ_1) and a chiral multiplet $(V_{5,6}, \lambda_2)$
- We assign parities under the first orbifolding $(x_5, x_6) \rightarrow (-x_5, -x_6)$
- $$\begin{aligned} PV_\mu(x, -x_5, -x_6)P^{-1} &= +V_\mu(x, x_5, x_6) \\ PV_{5,6}(x, -x_5, -x_6)P^{-1} &= -V_{5,6}(x, x_5, x_6) \end{aligned}$$
- Choosing $P = I$ we get
- $$\begin{aligned} V_\mu^{(-m,-n)} = +V_\mu^{(m,n)} &= +V_\mu^{(m,n)\dagger}, \\ V_{5,6}^{(-m,-n)} = -V_{5,6}^{(m,n)} &= +V_{5,6}^{(m,n)\dagger}. \end{aligned}$$

Gauge Breaking

- We also need to break the $SU(5)$ gauge theory to the standard model
- We use another orbifolding so our orbifold is now $\mathbb{T}^2/(\mathbb{Z}_2 \times \mathbb{Z}'_2)$
- We require that

$$P_{SM} V_\mu(x, -x_5 + \pi R_1/2, -x_6) P_{SM}^{-1} = +V_\mu((x, x_5 + \pi R_1/2, x_6)$$

- $$P_{SM} = \begin{pmatrix} +1 & 0 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$
- Gauge bosons belonging to the standard model have positive parity

Doublet-Triplet splitting

- We can choose a positive parity for the Higgs under both orbifoldings
- $$P_{SM}H(x, -x_5 + \pi R_1/2, -x_6) = H(x, x_5 + \pi R_1/2, x_6)$$
- This results in a light doublet and the coloured triplet becomes heavy

- $$w_{up} = \frac{1}{\Lambda^{1/2}} H_5 T_3 T_3 + \frac{\theta\theta''}{\Lambda^{5/2}} H_5 T_2 T_3 + \frac{\theta^2\theta^{2''}}{\Lambda^{9/2}} H_5 T_2 T_2 + \frac{\theta^3\theta''}{\Lambda^{9/2}} H_5 T_1 T_3 + \frac{\theta^6}{\Lambda^{13/2}} H_5 T_1 T_2 + \frac{\theta^3\theta^{3''}}{\Lambda^{13/2}} H_5 T_1 T_2 + \frac{\theta^7\theta''}{\Lambda^{17/2}} H_5 T_1 T_1 + \frac{\theta^4\theta^{4''}}{\Lambda^{17/2}} H_5 T_1 T_1 + \frac{\theta\theta^{7''}}{\Lambda^{17/2}} H_5 T_1 T_1$$

- $$w_{down} = \frac{1}{\Lambda^2} H_{\overline{5}} (F\varphi_T)'' T_3 + \frac{\theta^2}{\Lambda^4} H_{\overline{5}} (F\varphi_T)' T_2 + \frac{\theta^4}{\Lambda^6} H_{\overline{5}} (F\varphi_T)'' T_1 + \frac{\theta\theta^{3''}}{\Lambda^6} H_{\overline{5}} (F\varphi_T) T_1 + \frac{\theta\theta''}{\Lambda^4} H_{\overline{5}} (F\varphi_T)'' T_2 + \frac{\theta^3\theta''}{\Lambda^6} H_{\overline{5}} (F\varphi_T)' T_1 + \frac{\theta^2\theta^{2''}}{\Lambda^6} H_{\overline{5}} (F\varphi_T)'' T_1$$

- $$w_\nu = \frac{y^D}{\Lambda^{1/2}} H_5 (NF) + (x_a \xi + \tilde{x}_a \tilde{\xi})(NN) + x_b(\varphi_S NN)$$

VEVs

- We get a suppression factor $s = \frac{1}{\sqrt{\pi R \Lambda}}$
- $$\frac{\langle \varphi_T \rangle}{\Lambda} = (v_T, 0, 0), \quad \frac{\langle \varphi_S \rangle}{\Lambda} = (v_S, v_S, v_S), \quad \frac{\langle \xi \rangle}{\Lambda} = u$$
$$\frac{\langle \theta \rangle}{\Lambda} = t, \qquad \frac{\langle \theta'' \rangle}{\Lambda} = t''$$
- we assume $t \approx t'' \approx s \approx O(\lambda)$ with $\lambda \equiv 0.22$

Mass matrices

- $$m_u = \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} \lambda v_u^0$$

- $$m_d = \begin{pmatrix} \lambda^4 & \dots & \dots \\ \lambda^4 & \lambda^2 & \dots \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_T \lambda v_d^0$$

- $$m_e = \begin{pmatrix} st^3 + st''^3 & st^2 t'' & st t''^2 \\ \dots & st & st'' \\ \dots & \dots & 1 \end{pmatrix} v_T s v_d^0 = \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^4 \\ \dots & \lambda^2 & \lambda^2 \\ \dots & \dots & 1 \end{pmatrix} v_T \lambda v_d^0$$

Neutrinos

- $$m_\nu = \frac{1}{3a(a+b)} \begin{pmatrix} 3a+b & b & b \\ b & \frac{2ab+b^2}{b-a} & \frac{b^2-ab-3a^2}{b-a} \\ b & \frac{b^2-ab-3a^2}{b-a} & \frac{2ab+b^2}{b-a} \end{pmatrix} \frac{s^2(v_u^0)^2}{\Lambda}$$

- $$a \equiv \frac{2x_a u}{(y^D)^2}, b \equiv \frac{2x_b v_s}{(y^D)^2}$$

- $$U^T m_\nu U = \mathrm{diag}(m_1, m_2, m_3)$$

- $$U = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Summary

- Start with a 6d theory with full 6d spacetime symmetry
- Orbifold extra dimensions to leave an A_4 symmetry and 4d spacetime symmetry
- A SU(5) theory on the Orbifold making use of the A_4 as a family symmetry
- Orbifolding solves the Doublet-triplet splitting problem
- Produces tri-bimaximal neutrino mixing
- Still need to incorporate a Georgi-Jarlskog mechanism

References

Asaka, Buchmüller, Covi. Gauge unification in six-dimensions. hep-ph/0108021

Altarelli,Feruglio and Hagedorn. A SUSY SU(5) A_4 model. arXiv:0802.0090

Altarelli, Feruglio and Lin. Tri-bimaximal Neutrino Mixing from Orbifolding. hep-ph/0610165