Discrete anomalies

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What are discrete anomalies?

- Ibáñez & Ross (PLB 260 , NPB 368): Abelian discrete symmetries $\mathcal{Z}_N$
- embed $\mathcal{Z}_N \subset U(1)$
- $U(1)$ is gauged
- possible risk of $U(1)$ being anomalous

- $U(1)$ level: strong anomaly conditions at high energies
- $\mathcal{Z}_N$ level: weaker "discrete anomaly conditions" after $U(1)$ is broken

Abelian anomaly conditions I

$\Rightarrow$ assume gauge structure: $U(1) \times \mathrm{SM}$

$$\boxed{U(1) \;\supset\; \mathcal{Z}_N}$$

gauge symmetry | discrete symmetry

A N O M A L I E S

$$SU(3)_C - SU(3)_C - U(1) \qquad\qquad SU(3)_C - SU(3)_C - \mathcal{Z}_N$$

$$SU(2)_W - SU(2)_W - U(1) \qquad\qquad SU(2)_W - SU(2)_W - \mathcal{Z}_N$$

$$\mathrm{Gravity} - \mathrm{Gravity} - U(1) \qquad\qquad \mathrm{Gravity} - \mathrm{Gravity} - \mathcal{Z}_N$$

Abelian anomaly conditions II

- formulate anomaly equations in terms of $U(1)$ charges z_i

$$\text{structure}: \qquad \sum_i z_i \;=\; 0$$

- insert discrete charges q_i instead of $U(1)$ charges $\boxed{z_i \;=\; q_i \bmod N}$

$$\sum_i q_i \;=\; 0 \bmod N$$

- separate light and heavy fermions ($\mathcal{Z}_N$ invariant mass term)

$$\sum_{i=\text{light}} q_i + \underbrace{\sum_{i=\text{heavy}} q_i}_{0 \bmod N \text{ or } \frac{N}{2}} \;=\; 0 \bmod N$$

$$\Longrightarrow \quad \sum_{i=\text{light}} q_i \;=\; 0 \bmod N \text{ or } \frac{N}{2}$$

Discrete family symmetries $\mathcal{G}$ are non-Abelian

$\Rightarrow$ embed $\mathcal{G}$ into a continuous gauge group G

G	$\supset$	$\mathcal{G}$
$SU(3)$		$\mathcal{PSL}_2(7),\ \mathcal{Z}_7 \rtimes \mathcal{Z}_3,\ \Delta(3n^2),\ \Delta(6n^2)$
$SO(3)$		$\mathcal{S}_4,\ \mathcal{A}_4,\ \mathcal{S}_3,\ \mathcal{D}_n$
$SU(2)$		$\mathcal{T}',\ \mathcal{Q}_{2n}$

$\mathcal{A}_4 \times \mathrm{SM}$

- high energy origin: $SO(3) \times \mathrm{SM}$
- possible anomaly: $SO(3) - SO(3) - U(1)_Y$

$$\sum_{\boldsymbol{\rho}} \ell(\boldsymbol{\rho}) \cdot Y(\boldsymbol{\rho}) = 0$$

Dynkin indices for $SO(3)$ representations

$$\mathrm{Tr}\left(\left\{T_a^{[\boldsymbol{\rho}]}\,,\,T_b^{[\boldsymbol{\rho}]}\right\}\right) \;=\; \ell(\boldsymbol{\rho})\,\delta_{ab}$$

$$T_a^{[\boldsymbol{\rho}]} \;=\; SO(3)\ \text{generators}$$

Irreps $\boldsymbol{\rho}$ of $SO(3)$	$\ell(\boldsymbol{\rho})$
3	1
5	5
7	14
9	30
11	55

Discrete indices for $\mathcal{A}_4$ representions

- $\mathcal{A}_4$ representations $\mathbf{r_i}$ can originate from different $SO(3)$ representations $\boldsymbol{\rho}$

$$\boldsymbol{\rho} \longrightarrow \sum_i \mathbf{r_i}$$

- define "discrete indices" $\widetilde{\ell}(\mathbf{r_i})$ such that:

$$\ell(\boldsymbol{\rho}) \quad = \quad \sum_i \widetilde{\ell}(\mathbf{r_i}) \mod N_\ell$$

for all irreps $\boldsymbol{\rho}$ of $SO(3)$

Embedding $\mathcal{A}_4$ into $SO(3)$

discrete indices of $\mathcal{A}_4$:

Irreps $\mathbf{r_i}$ of $\mathcal{A}_4$	$\widetilde{\ell}(\mathbf{r_i})$ ($N_\ell = 12$)
$\mathbf{1}$	0
$\mathbf{1}'$	2
$\overline{\mathbf{1}'}$	2
$\mathbf{3}$	1

$\boldsymbol{\rho}$	$\sum_i \mathbf{r_i}$	$\ell(\boldsymbol{\rho})$	$\sum_i \widetilde{\ell}(\mathbf{r_i})$
$\mathbf{3}$	$\mathbf{3}$	1	1
$\mathbf{5}$	$\mathbf{1}' + \overline{\mathbf{1}'} + \mathbf{3}$	5	5
$\mathbf{7}$	$\mathbf{1} + 2\cdot\mathbf{3}$	14	2
$\mathbf{9}$	$\mathbf{1} + \mathbf{1}' + \overline{\mathbf{1}'} + 2\cdot\mathbf{3}$	30	6
$\mathbf{11}$	$\mathbf{1}' + \overline{\mathbf{1}'} + 3\cdot\mathbf{3}$	55	7

$\mathcal{A}_4$ anomaly condition

- high energy anomaly equation

$$\sum_{\boldsymbol{\rho}} \ell(\boldsymbol{\rho}) \cdot Y(\boldsymbol{\rho}) \ = \ 0$$

- replace Dynkin indices by discrete indices $\left[\ell(\boldsymbol{\rho}) \ = \sum_i \widetilde{\ell}(\mathbf{r_i}) \bmod N_\ell\right]$

$$\sum_i \widetilde{\ell}(\mathbf{r_i}) \cdot Y(\mathbf{r_i}) \ = \ 0 \bmod N_\ell$$

- separate light and heavy fermions ($\mathcal{G}$ invariant mass term)

$$\sum_{i=\text{light}} \widetilde{\ell}(\mathbf{r_i}) \cdot Y(\mathbf{r_i}) \ + \underbrace{\sum_{i=\text{heavy}} \widetilde{\ell}(\mathbf{r_i}) \cdot Y(\mathbf{r_i})}_{0 \bmod N_\ell} \ = \ 0 \bmod N_\ell$$

$$\Longrightarrow \quad \sum_{i=\text{light}} \widetilde{\ell}(\mathbf{r_i}) \cdot Y(\mathbf{r_i}) \ = \ 0 \bmod N_\ell$$

An $\mathcal{A}_4$ example

– all hypercharges are integer

– hypercharge normalization $Y_Q = 1$

$$\sum_{i=\text{light}} \widetilde{\ell}_i\, Y_i \;=\; 0 \bmod 12$$

- maybe $\mathcal{A}_4$ is a special feature of the lepton sector
- quarks transform as singlets under $\mathcal{A}_4$
- only leptons contribute to discrete anomaly

$$L \;\sim\; \mathbf{3}\,, \qquad E^c \;\sim\; \mathbf{1} + \mathbf{1}' + \overline{\mathbf{1}'}\,, \qquad N^c \;\sim\; \mathbf{3}$$

$$2 \times 1 \cdot (-3) \;+\; 4 \cdot 6 \;+\; 1 \cdot 0 \;=\; 18 \;\neq\; 0 \bmod 12$$

all the (nasty) details:

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