Synoptic Exam Course Work – BSc & MPhys

An excellent way in which to improve your exam technique is to place yourself on the other side of the process – that is to think about actually setting an exam paper. Further more as you move on from undergraduate studies to other careers you may find yourself setting exam problems and it is good practice to have thought about the process.

For these reasons we have decided that as the assessed course work part of the Synoptic Exam course (worth 20% of the marks on the course) we will require you to write one section A and one section B style question, with full solutions, as if for this Synoptic course.

Here are the relevant bits of the School's staff handbook on examination preparation:

1.3 Guidelines for Producing Examination Papers

1.4 Format of exam papers

The section A + section B Format is now the default for all School examinations. Section A is expected to take roughly 40 minutes, with straightforward questions spanning all the basics of the syllabus and requiring short answers. There should be between 5 and 10 questions. The marks for each question may vary, but the total mark for section A should be 20. One of the purposes of including section A is to ensure that our students have met the minimum learning outcomes which cover the whole syllabus so please remember this when drafting your questions.

Section B should contain more demanding questions and should have a choice of 2 out of 4 questions, each to be worth 20 marks and be answered in roughly 40 minutes.

1.5 Designing good questions

Programme regulations require students to achieve a minimum of 40% in all core units and this must be considered in designing your paper. Question papers need to be able to test a wide range of ability whilst being absolutely assured of including parts of questions that will allow weaker students to pass by demonstrating sufficient knowledge/ability to meet the units learning outcomes. It is equally important to ensure that the questions make provision to really test the first class students; this in turn prevents inflation of the average unit mark. Questions should be designed in such a way that they comprise small independent parts so that failure to answer the first part does not make it impossible to answer the whole question.

1.6 Solution and the marking scheme

Worked solutions must be provided by the published deadline and should clearly indicate the minimal student answer that would gain full marks. The solutions must include a detailed marking scheme, although the allocation of marks shown on the paper can be coarser than this. The marking scheme should be sufficiently detailed so that no part of an answer will have more than 2½ marks assigned to it. You are strongly encouraged to use a scale involving half marks. The model answers must indicate clearly the parts of the question that are `bookwork', `discussed in lectures', `similar to problems on problem sheets', `original problems' etc. Descriptive solutions should clearly indicate all points that the candidate is expected to make, with a mark or marks assigned for each point.

Your Section A Question

Your section A question should be worth 4 marks (ie a fifth of a full section A) – you should be testing core knowledge in first and second year core units of the BSc/MPhys program. Provide a typed solution showing precisely where the marks would be allocated and comment on the degree to which the material has been covered in the course.

Use last years examination paper as a guide – I will also provide some mock questions through the lectures to provide further aid.

Here, when I mark your work, I shall be looking for a question that picks an important piece of the first two years syllabus that every student could reasonably be expected to know when they graduate. You will be marked on your choice of the level and appropriateness of the question, whether it is clearly worded and the clearness of the solution.

Your Section B Question

Your section B question should be worth 20 marks. Again base the problem on the most important concepts from the core first and second year courses (not third year even in the case of MPhys students).

An excellent section B question will contain short questions testing core knowledge that all students should be expected to know at graduation (these should be worth roughly 8 marks to meet the 40% pass mark)… it would also require knowledge from more than one course to test synthesis of knowledge… it would contain some unseen problem solving element… possibly using an important mathematical techique such as binomial expansion… and finally have a (one or two mark) kick in its tail to challenge the top first class students. In practice including all these elements is challenging (see how you think we did in last years exam!) but you should aim to tick several of these boxes in your question.

Again you must provide a typed solution showing precisely where the marks would be allocated and comment on the degree to which the material has been covered in the course. You will be marked on your choice of the level and appropriateness of the question, whether it is clearly worded and the clearness of the solution.

Presenting Your Questions and Solutions

Please type both your questions and answers – we will not bother here with the precise fonts etc appropriate to a University of Southampton exam but please make it legible and base your style on that of the exams you have taken. For example use bold for vectors!

Hand in Date

Please submit your course work to the school office by the end of week 9 of the semester – ie by noon on 1st April.

Sample Problems & Solutions

Below are two of the problems from last years exam with solutions in the expected format:

A1. Show, starting from the definition of acceleration as the rate of change of velocity, that if a particle moving at speed u is subject to constant acceleration a in the direction of its motion, that after it has travelled a distance s, the magnitude of its velocity is given by

$$v^2 = u^2 + 2as$$

[3]

Solution: course work - seen in first year lectures.

$$\begin{aligned} \text{acceleration} = a &= \tfrac{dv}{dt} \\ &= \tfrac{dv}{dx}\tfrac{dx}{dt} \qquad [1 \;\; \text{mark}] \\ &= v\tfrac{dv}{dx} \qquad [1/2 \;\; \text{mark}] \end{aligned}$$

Integrating gives

$$\frac{1}{2}v^2 - \frac{1}{2}u^2 = as$$

with u a constant of integration. [1 mark]

Rearranging gives

$$v^2 = u^2 + 2as \qquad [1/2 \;\; \text{mark}]$$

B4. Consider a particle of mass m moving in one dimension in a potential, $V(x)$, with a minimum at the position x_0.

(a) Write down the generic form of the Taylor expansion of the potential about the minimum keeping the leading two terms in the expansion [4]

(b) Remembering that force, **F** is given in terms of a potential, V, by

$$\mathbf{F} = -\nabla V$$

Derive the equation of motion for the particle making small motions about the potential minimum and show that it has simple harmonic form with angular frequency given by

$$\omega^2 = \frac{d^2V}{dx^2}|_{x_0}$$

[7]

(c) Explain why sine and cosine waves are so prevalent in nature. [3]

(d) An example of such a one dimensional potential is this: the particle has charge $+q$ and is situated between two other charges of $+q$ separated by a distance $2a$. Write down the potential it experiences and hence compute the angular frequency of the particle's oscillations about the minimum of the potential. [6]

Solution:

(a) (First year maths taught knowledge) The basic Taylor expansion is given by

$$V(x_0+\delta x) = V(x_0) + \delta x\ \frac{dV}{dx}|_{x=x_0} + \frac{1}{2}\delta x^2\ \frac{d^2V}{dx^2}|_{x=x_0} + ... \qquad [2 \text{ marks}]$$

Since x_0 is the minimum

$$\frac{dV}{dx}|_{x=x_0} = 0 \qquad [1 \text{ mark}]$$

Leaving

$$V(x_0+\delta x) = V(x_0) + \frac{1}{2}\delta x^2\ \frac{d^2V}{dx^2}|_{x=x_0} + ... \qquad [1 \text{ mark}]$$

(b) (New problem solving requiring knowledge of SHM)
In one dimension for motion δx

$$F = -\frac{d}{d\ \delta x}V \qquad [1 \text{ marks}]$$

Subing in the potential from (a)

$$F = -\delta x\ \frac{d^2V}{d\ \delta x^2}|_{x=x_0} + ... \qquad [2 \text{ marks}]$$

Now

$$F = m\frac{d^2\delta x}{dt^2} \qquad [1 \text{ mark}]$$

So

$$\frac{d^2\delta x}{dt^2} = \frac{1}{m}\frac{d^2V}{d\ \delta x^2}|_{x=x_0}\delta x \qquad [1 \text{ mark}]$$

SHM is described by $\frac{d^2\delta x}{dt^2} = -\omega^2\delta x$ [1 mark]

Giving the solution in the question (ooops - we had a typo because we dropped the $1/m$ in the question!) [1 mark]

(c) (Not explicitly taught)

Most physical systems can be well described by small motion about minimum energy configurations. [1 mark]

We have just shown in (b) that such small motion may be considered simple harmonic no matter the potential V. [1 mark]

The solution to the equation

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

is $x = \sin(\omega t + \phi)$ so sine and cosine waves are the natural output. [1 mark]

(d) (Electromagnetism problem solving using previous given results in question) For the central charge (positioned at origin) the potential is

$$V(x) = \frac{q^2}{4\pi\epsilon_0(a+x)} + \frac{q^2}{4\pi\epsilon_0(a-x)} \qquad [2 \text{ marks}]$$

Now compute first spatial derivative

$$\frac{dV}{dx} = \frac{q^2}{4\pi\epsilon_0}\left(\frac{-1}{(a+x)^2} + \frac{1}{(a-x)^2}\right)$$

which vanishes at $x = 0$ [1 mark].

The second derivative is

$$\frac{d^2V}{dx^2} = \frac{q^2}{4\pi\epsilon_0}\left(\frac{2}{(a+x)^3} + \frac{2}{(a-x)^3}\right) \qquad [2 \text{ marks}]$$

The angular frequency is therefore (evaluating at $x = 0$)

$$\omega_0 = \sqrt{\frac{q^2}{4\pi\epsilon_0 m}\frac{4}{a^3}} \qquad [1 \text{ mark}]$$