Discrete symmetries and their breaking in the 3HDM scalar sector

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based on:

Ivanov, Vdovin, EPJC 73, 2309 (2013);
Degee, Ivanov, Keus, JHEP 1302, 125 (2013);
Ivanov, Nishi, arXiv:1410.6139.

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Outline

What this talk is about

N-Higgs-doublet models, NHDM, is a broad and conservative class of bSM models in which we just assume that Higgs doublets come in "generations": ϕ_i, $i = 1, \ldots, N$.

Motivations: rich scalar sector, insights into the flavor puzzle, novel forms of CP-violation, astroparticle effects, etc. $\rightarrow \mathcal{O}(10^3)$ works based on various forms of NHDM.

What this talk is about

I am not going to:

- promote any specific bSM model based on several Higgs doublets,
- or give detailed predictions for the LHC or astroparticle observables.

My interest is to systematically investigate the entire class of models, at least in the scalar sector.

I will focus on 3HDM and will explore what's possible, symmetry-wise, in the scalar sector of 3HDM.

Even if you are not familiar with this subject, no problem: take it as an example of somewhat unusual application of the group theory to physics.

Scalar sector in NHDM

SM, 2 parameters: $V = -\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2$.

2HDM, 14 parameters:

$$\begin{aligned} V &= -\frac{1}{2}\left[m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) + m_{12}^2(\phi_1^\dagger\phi_2) + m_{12}^{2\,*}(\phi_2^\dagger\phi_1)\right] \\ &+\frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ &+\left\{\left[\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2) + \lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)\right](\phi_1^\dagger\phi_2) + \mathrm{h.c.}\right\}. \end{aligned}$$

NHDM, $N^2(N^2+3)/2$ parameters:

$$V = Y_{ab}(\phi_a^\dagger\phi_b) + Z_{abcd}(\phi_a^\dagger\phi_b)(\phi_c^\dagger\phi_d)\,,$$

No chance to qualitatively understand the phase diagram by simply scanning the entire parameter space!

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Symmetries in multi-Higgs models

- A good way to see structures in multi-parametric models is to understand possible symmetry classes and track their consequences.
- $\mathcal{L}_{\rm bSM} \to$ impose symmetries $\to$ get structures $\to$ study phenomenological consequences (CP, FCNC, astroparticle, flavour, etc).
- Classic papers on 3HDM with symmetries: *CP*-violating $\mathbb{Z}_2 \times \mathbb{Z}_2$ *[Weinberg, 1979]*, *CP*-conserving $\mathbb{Z}_2 \times \mathbb{Z}_2$ *[Branco, 1980]*, S_3 *[Pakvasa, Sugawara, 1978]*, A_4 *[Ma, Rajasekaran, 2001]*, $\Delta(27)$ *[Branco, Gerard, Grimus, 1984]*, + many dozens of more recent works.
- Systematic exploration of all symmetry-related issues in a model with N doublets is a challenging but rewarding undertaking. Understanding how symmetries work in bSM models will support the mainstream model-building activity.

Symmetries in multi-Higgs models

I will focus on discrete symmetries in the scalar sector of 3HDM, with some discussion of NHDM.

Both Higgs-family transformations $\phi_i \mapsto U_{ij}\phi_j$ and generalized-CP (GCP) transformations $\phi_i \mapsto U_{ij}\phi_j^*$ transformations will be considered.

Main questions:

- Which Higgs-family symmetry groups G_{HF} can the scalar sector have?
- What are the CP-consequences of each G_{HF}?
- How do these symmetries break upon minimization of the potential?

Technical remarks

- Since we work only with the scalar sector, unitary transformations are considered up to overall rephasing:
  $U_{ij} \in U(N)/U(1) \simeq SU(N)/\mathbb{Z}_N = PSU(N)$.

  The most important example is $\Delta(27) \in U(3)$ in 3HDM,

$$\Delta(27) = \langle a_3,\, b\rangle\,, \quad a_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}\,, \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\,,$$

  becomes $\Delta(27)/\mathbb{Z}_3 \simeq \mathbb{Z}_3 \times \mathbb{Z}_3 \in PSU(3)$.

- Important difference between imposing and deriving symmetry group: below, G-symmetric 3HDM means that V is not only G-invariant, but also does not have any other symmetry beyond G.

  G represents the full symmetry content of the model.

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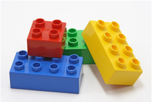
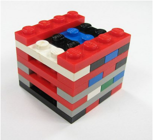
Finding discrete symmetries in the 3HDM scalar sector

"Abelian LEGO" strategy

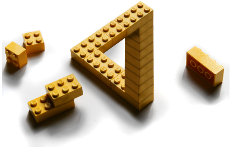
Step 1: find all possible discrete abelian groups A_i; any allowed G can have only those abelian subgroups. These are "LEGO bricks" with which we will build a non-abelian model.



Step 2: build G by combining various A_i but avoid producing abelian groups not in the list!



Step 3: for each G built, check that it fits $PSU(3)$ and that it does not automatically produce any higher symmetry.



Step 1: Abelian symmetry groups

We developed a systematic procedure (Smith normal form technique) which gives all possible rephasing symmetry groups for any bSM model with any set of complex fields.

In the specific case of the 3HDM scalar sector, we include one extra abelian subgroup of $PSU(3)$ which is not rephasing group. The final list is

$$\mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_4, \quad \mathbb{Z}_2 \times \mathbb{Z}_2, \quad \mathbb{Z}_3 \times \mathbb{Z}_3\,.$$

It is complete: imposing any other finite abelian symmetry group on the potential unavoidably leads to continuous symmetry group.

Note that the orders of these groups divide only two primes: 2 and 3.

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Step 2: Group-theoretic part

- Any finite (non-abelian) G must contain only these abelian subgroups,
- $\Rightarrow$ by Cauchy's theorem, its order $|G| = 2^a 3^b$,
- $\Rightarrow$ by Burnside's $p^a q^b$ theorem, G contains a normal abelian subgroup A

$$g^{-1} A g = A \quad \forall g \in G\,.$$

- At this point, we cannot yet restrict G/A.
- With further group theory, we proved a stronger statement: G contains a normal maximal abelian subgroup, which has remarkable consequences.

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Consequences of a normal maximal abelian subgroup

- If A is normal in G, then $g^{-1}Ag = A$, so g acts on elements of A by some group-preserving permutation (automorphism of A).
- So, for every $g \in G$ we get an automorphism $\in Aut(A)$. We get a map $f : G \to Aut(A)$.
- $Ker\, f = C_G(A)$ (centralizer of A in G): contains all elements g which act trivially on A: $g^{-1}ag = a$ for all $a \in A$.
- If A is maximal abelian, then such g must belong to A itself. Then A is self-centralizing, $Ker\, f = A$:

$$G/Ker\, f = G/A \subseteq Aut(A),$$

and G can be constructed as an extension of A by a subgroup of $Aut(A)$.

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Automorphism groups

$$G = \text{extension of } A \text{ by } P\,, \quad P \subseteq Aut(A)\,.$$

Overview of possibilities:

A	$Aut(A)$	"usable" subgroups P
$\mathbb{Z}_2$	$\{1\}$	—
$\mathbb{Z}_3$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\mathbb{Z}_4$	$\mathbb{Z}_2$	$\mathbb{Z}_2$
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$GL_2(2) \simeq S_3$	$\mathbb{Z}_2,\ \mathbb{Z}_3,\ S_3$
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$GL_2(3)$	$\mathbb{Z}_2,\ \mathbb{Z}_4$

Step 3: Constructing G by extensions

Example: $A = \mathbb{Z}_4$. Then $Aut(\mathbb{Z}_4) = \mathbb{Z}_2$, so G is extension of $\mathbb{Z}_4$ by $\mathbb{Z}_2$.

There are several possibilities.
(1) extensions which lead to larger abelian groups ($\mathbb{Z}_8$, $\mathbb{Z}_4 \times \mathbb{Z}_2$) are immediately excluded;

(2) dihedral group D_4, the symmetry group of the square.

$$D_4 = \langle a, b \rangle \text{ with conditions } a^4 = 1,\ b^2 = 1,\ ab = ba^3\,.$$

If $a = \mathrm{diag}(i, -i, 1)$, then

$$b = \begin{pmatrix} 0 & e^{i\delta} & 0 \\ e^{-i\delta} & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \text{with arbitrary } \delta.$$

Finally, we construct a generic D_4-invariant potential and check that it has no other symmetry.

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We construct the Q_4-invariant potential and find that it is unavoidably invariant under a $U(1)$ group.

We conclude that Q_4 alone cannot be a symmetry group of potential.

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3HDM scalar symmetries

Discrete non-abelian G_{HF}'s allowed in the 3HDM scalar sector:

$$G_{HF} = S_3\,, \quad D_4\,, \quad A_4\,, \quad S_4\,, \quad \Delta(54)/\mathbb{Z}_3\,, \quad \Sigma(36)\,.$$

This list is complete: trying to impose any other finite Higgs-family symmetry group on the 3HDM potential will unavoidably lead to a continuous symmetry.

Checking CP properties

Explicitly CP-violating 3HDM:

$$G = \mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_2 \times \mathbb{Z}_2\,, \quad S_3\,, \quad \Delta(54)/\mathbb{Z}_3\,.$$

Explicitly CP-conserving 3HDM
($*$ indicates a group generated by a GCP transformation):

$$\begin{aligned} G \;=\; & \mathbb{Z}_2^*, \quad \mathbb{Z}_2 \times \mathbb{Z}_2^*, \quad \mathbb{Z}_4^*, \quad \mathbb{Z}_3 \rtimes \mathbb{Z}_2^*, \quad \mathbb{Z}_4 \rtimes \mathbb{Z}_2^*, \quad \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2^*\,, \\ & S_3 \times \mathbb{Z}_2^*, \quad D_4 \times \mathbb{Z}_2^*, \quad A_4 \rtimes \mathbb{Z}_2^*, \quad S_4 \times \mathbb{Z}_2^*, \\ & (\Delta(54)/\mathbb{Z}_3) \rtimes \mathbb{Z}_2^*, \quad \Sigma(36) \times \mathbb{Z}_2^*\,. \end{aligned}$$

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Breaking discrete symmetries

Symmetry breaking patterns in NHDM

Consider NHDM with scalar symmetry group G. After EWSB, we get a neutral vacuum with a certain vev alignment $\langle\phi_i^0\rangle = v_i e^{i\xi_i}/\sqrt{2}$ invariant under a residual symmetry group $G_v \subseteq G$.

There situations are possible:

- symmetry is conserved: $G_v = G$;
- symmetry is partially broken: $\{e\} \subset G_v \subset G$;
- symmetry is completely broken: $G_v = \{e\}$.

The goal: for each G, establish its symmetry breaking patterns.

Phenomenology depends a lot on how much of the original symmetry is broken! Quark sector: NHDM with symmetry group G can lead to viable quark masses and CKM only if G is broken completely in the space of "active" doublets *[Leurer, Nir, Seiberg, 1993; Gonzalez Felipe, Ivanov, Nishi, Serodio, Silva, 2014]*.

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Minimization of 3HDM potentials

Some remarks:

- For small groups G lots of free parameters make any symmetry breaking pattern possible.
- Groups such as $Z_3 \rtimes \mathbb{Z}_2^*$ and $Z_4 \rtimes \mathbb{Z}_2^*$ have very few phase-sensitive terms, and it helps to derive conclusions on symmetry breaking.

  For example, $Z_4 \rtimes \mathbb{Z}_2^*$-symmetric 3HDM is $V = V_0 + V_{ph}$, where V_0 depends only on $|\phi_i|^2$ and

$$V_{ph} = \lambda(\phi_2^\dagger\phi_1)(\phi_3^\dagger\phi_1) + \lambda'(\phi_3^\dagger\phi_2)^2 + h.c.$$

  with real λ, λ'. It is symmetric under $a_4 = \mathrm{diag}(1, i, -i)$ and CP. Parametrizing vevs as $(v_1,\ v_2 e^{i\xi_2},\ v_3 e^{i\xi_3})$ and differentiating V, one gets rigid phases such as $\xi_2 = -\xi_3 = \pi/4$. As a result, the full breaking of $Z_4 \rtimes \mathbb{Z}_2^*$ is impossible.

Minimization of 3HDM potentials

For large symmetry groups, the potential has trivial quadratic part and very few terms in the quartic part. The global minimum can be found much more efficiently with a geometric method rather than the traditional sequence *[Degee, Ivanov, Keus, 2013]*.

The main idea is to rewrite the G-symmetric potential as a linear function of certain real variables:

$$V = -\frac{1}{2}m^2 v^2 + \frac{1}{4}v^4\left(\Lambda_0 + \Lambda_1 x_1 + \Lambda_2 x_2 + \cdots + \Lambda_k x_k\right).$$

Variables x_i do not depend on v; they reflect the relative structures in the vev alignment, and satisfy certain inequalities. Finding these inequalities defines the orbit space in the space of all x_i.

Once the shape of this orbit space is constructed, minimization of V becomes a trivial geometric exercise and can be done for all possible values of free parameters.

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Minimization of the "$\Delta(27)$" 3HDM

To illustrate this method, consider "*CP*-violating $\Delta(27)$" 3HDM (with the true symmetry group $\Delta(54)/\mathbb{Z}_3$):

$$\begin{aligned}
V_1 &= -m^2\left[\phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2+\phi_3^\dagger\phi_3\right]+\lambda_0\left[\phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2+\phi_3^\dagger\phi_3\right]^2\\
&+\frac{\lambda_1}{3}\left[(\phi_1^\dagger\phi_1)^2+(\phi_2^\dagger\phi_2)^2+(\phi_3^\dagger\phi_3)^2-(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)-(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3)-(\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1)\right]\\
&+\lambda_2\left[|\phi_1^\dagger\phi_2|^2+|\phi_2^\dagger\phi_3|^2+|\phi_3^\dagger\phi_1|^2\right]\\
&+\left(\lambda_3\left[(\phi_1^\dagger\phi_2)(\phi_1^\dagger\phi_3)+(\phi_2^\dagger\phi_3)(\phi_2^\dagger\phi_1)+(\phi_3^\dagger\phi_1)(\phi_3^\dagger\phi_2)\right]+h.c.\right),
\end{aligned}$$

which can be cast in the form

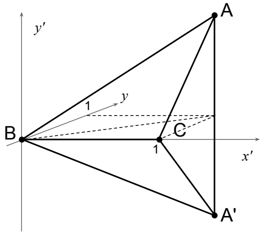
$$V_1=-\frac{1}{2}m^2v^2+\frac{1}{4}v^4\left(\Lambda_0+\Lambda_1x+\Lambda_1'x'+\Lambda_2y+\Lambda_2'y'\right),$$

with appropriately defined x, y, x', y'. With some algebra, one finds that

$$x=1\,,\quad 0\le y\le x'\le 1\,,\quad |y'|\le y\,.$$

Minimization of the "$\Delta(27)$" 3HDM

These conditions define a tetrahedron in the (x', y, y')-space. We need to minimize a linear function on the orbit space which lies inside this tetrahedron (but does not fill it completely).



The key observation: the four vertices belong to the orbit space $\to$ the global minimum can only be at those points.

Up to cyclic permutations,

$$A: \ (\omega, 1, 1), \quad A': \ (\omega^2, 1, 1), \quad B: \ (1, 0, 0),$$

and

$$C: \ (1, 1, 1), \quad (1, \omega, \omega^2), \quad (1, \omega^2, \omega).$$

Finding explicit conditions on parameters Λ is straightforward. No other vev can be the global minimum of this potential for any values of Λ's.

Minimization of the "$\Delta(27)$" 3HDM

Passing to larger groups is straightforward: just collapse the orbit space on the plane $y' = 0$, for "CP-conserving $\Delta(27)$", and further on the axis y, for $\Sigma(36)$.

In all three cases, the global minimum can only reside at these points. In the CP-conserving case, points A and A' realize the geometric-CP violation *[Branco, Gerard, Grimus, 1984]*. Remarkably, the same geometric phase persists even for explicitly CP-violating!

Since we now have all possible vev alignments, we can proceed with symmetry breaking patterns at each minimum.

Symmetry breaking in 3HDM

Results on strongest and weakest breaking of discrete symmetries in 3HDM, as well as on spontaneous *CP*-violation.

group	$\|G\|$	$\|G_v\|_{min}$	$\|G_v\|_{max}$	sCPv possible?
abelian	2, 3, 4, 8	1	$\|G\|$	yes
$\mathbb{Z}_3 \rtimes \mathbb{Z}_2^*$	6	1	6	yes
S_3	6	1	6	—
$\mathbb{Z}_4 \rtimes \mathbb{Z}_2^*$	8	2	8	no
$S_3 \times \mathbb{Z}_2^*$	12	2	12	yes
$D_4 \times \mathbb{Z}_2^*$	16	2	16	no
$A_4 \rtimes \mathbb{Z}_2^*$	24	4	8	no
$S_4 \times \mathbb{Z}_2^*$	48	6	16	no
CP-violating $\Delta(27)$	18	6	6	—
CP-conserving $\Delta(27)$	36	6	12	yes
$\Sigma(36)$	72	12	12	no

Symmetry breaking in 3HDM

- Spontaneous CP-violation is possible only for those Higgs-family groups G_{HF}, for which there exists an explicitly CP-violating model. If G_{HF} forbids explicit CP violation, it also forbids spontaneous CP-violation. Explicit and spontaneous CP violations come in pairs.
- When we break a discrete symmetry group, we have several degenerate minima. Usual expectation: all minima lie on a single G-orbit: one can link any pair of minima by a broken symmetry $\in G$. Then, the number of degenerate minima is equal to the length of the orbit

$$\ell = |G|/|G_v|\,.$$

In one case, this expectation breaks: $G = A_4 \rtimes \mathbb{Z}_2^*$, $|G| = 24$, breaks at $(\pm 1, \omega, \omega^2)$ to $G_v = \mathbb{Z}_3 \rtimes \mathbb{Z}_2^*$, $|G_v| = 6$, producing eight minima lying on two disjoint orbits.
- Unlike 2HDM, minima with different symmetry breaking can coexist in 3HDM and even be degenerate.

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Towards NHDM result: a possible line of attack

What prevents sufficiently large discrete groups from complete breaking? There must exist an upper bound n_{max} on the number of minima of the NHDM potential. Explicit calculations show that $n_{max} = 2$ for 2HDM and $n_{max} = 8$ for 3HDM. Therefore, groups with $|G| > n_{max}$ cannot break completely. The difficult question is to actually find n_{max} for general N.

When working in the space on bilinears $r_a = \phi_i^\dagger \lambda^a_{ij} \phi_j$, minimization of the NHDM potential can be cast in purely geometric terms. Search for the global minimum = search for contact points between two algebraic manifolds, the potential V and the orbit space.

If two algebraic manifolds in $\mathbb{R}^k$ of degrees m_1 and m_2 intersect, there must exist an upper bound on the number of connected components. For planar curves, it is $m_1 m_2$ (Bezout's theorem); we just need its analog for higher k.

Note that n_{max} depends on the algebraic order of the potential → G-symmetric higher-order terms might lead to stronger symmetry breaking than quadratic+quartic.

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Conclusions

- We investigated in full detail discrete symmetries and their breaking in 3HDM. All allowed symmetry groups are found, and for each group all symmetry breaking patterns are established. Interplay between Higgs-family symmetries and *CP*-violation is investigated.
- This study serves both as an input to specific 3HDM models and, with the peculiar regularities we observed, as a step towards general understanding of discrete symmetry breaking in the scalar sector of NHDM.